Accretion Flows
near Black Holes:
Effects of General Relativity

Vladimír Karas (Astronomical Institute, Prague)

`vladimir.karas@cuni.cz`

Some historical remarks

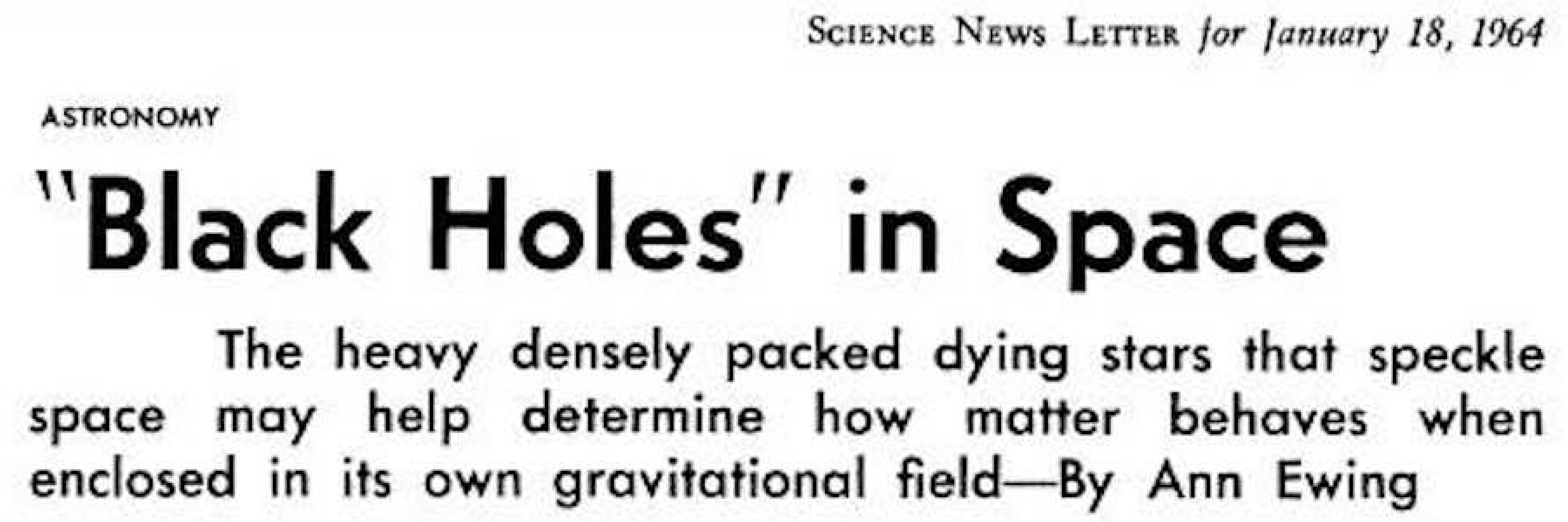
John Michell (1784): *Phil. Trans. Roy. Soc. Lond.*, LXXIV, 35

If there should really exist in nature any bodies whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us . . . we could have no information from sight; yet if any other luminous bodies should happen to revolve about them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones. . .

Pierre S. Laplace (1796): *“Exposition du Système du Monde”*

. . . the attractive force of a heavenly body could be so large that light could not flow out of it.

Some historical remarks



SCIENCE NEWS LETTER *for January 18, 1964*

ASTRONOMY

"Black Holes" in Space

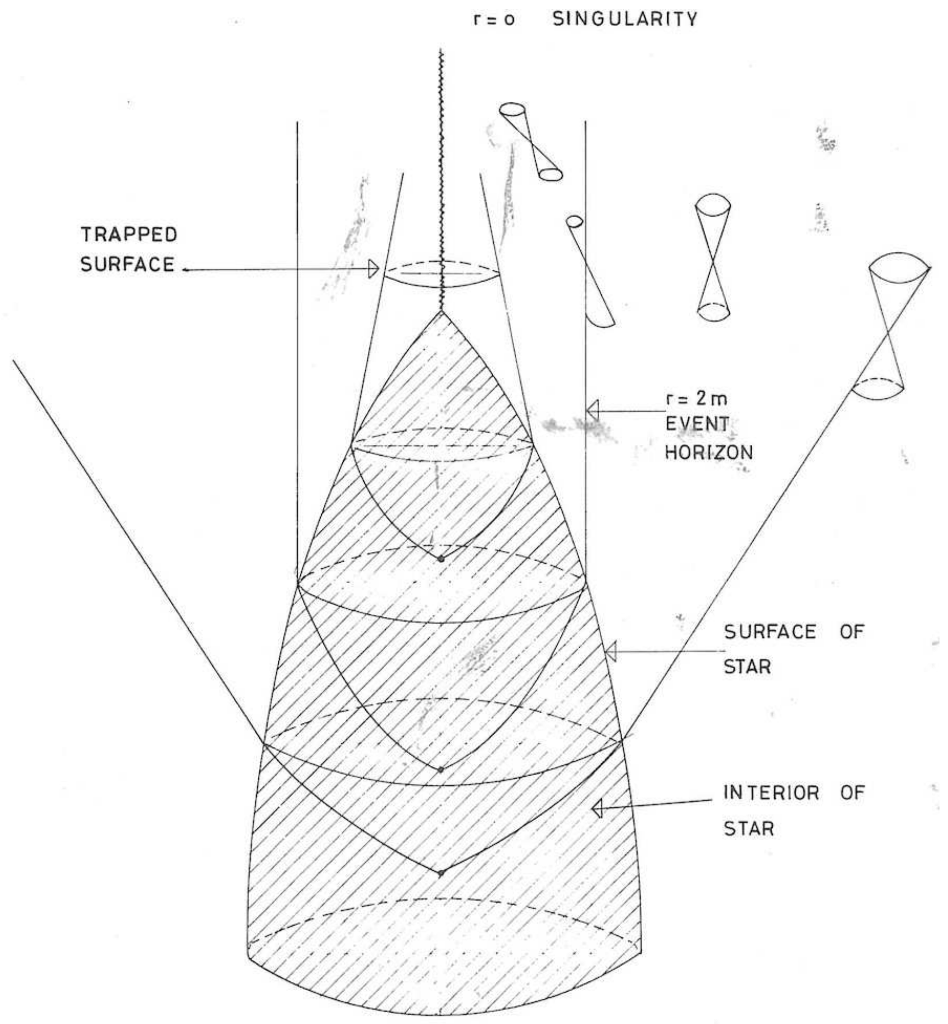
The heavy densely packed dying stars that speckle space may help determine how matter behaves when enclosed in its own gravitational field—By Ann Ewing

But . . .

. . . It is virtually impossible to prove unambiguously the presence of black-hole horizon by observing the electromagnetic signal from its presumed vicinity in a cosmic system – absence of radiation signal does not necessarily prove the absence of the surface of the body and the existence of the horizon.

(See Abramowicz et al. 2002, A&A, 396, L31)

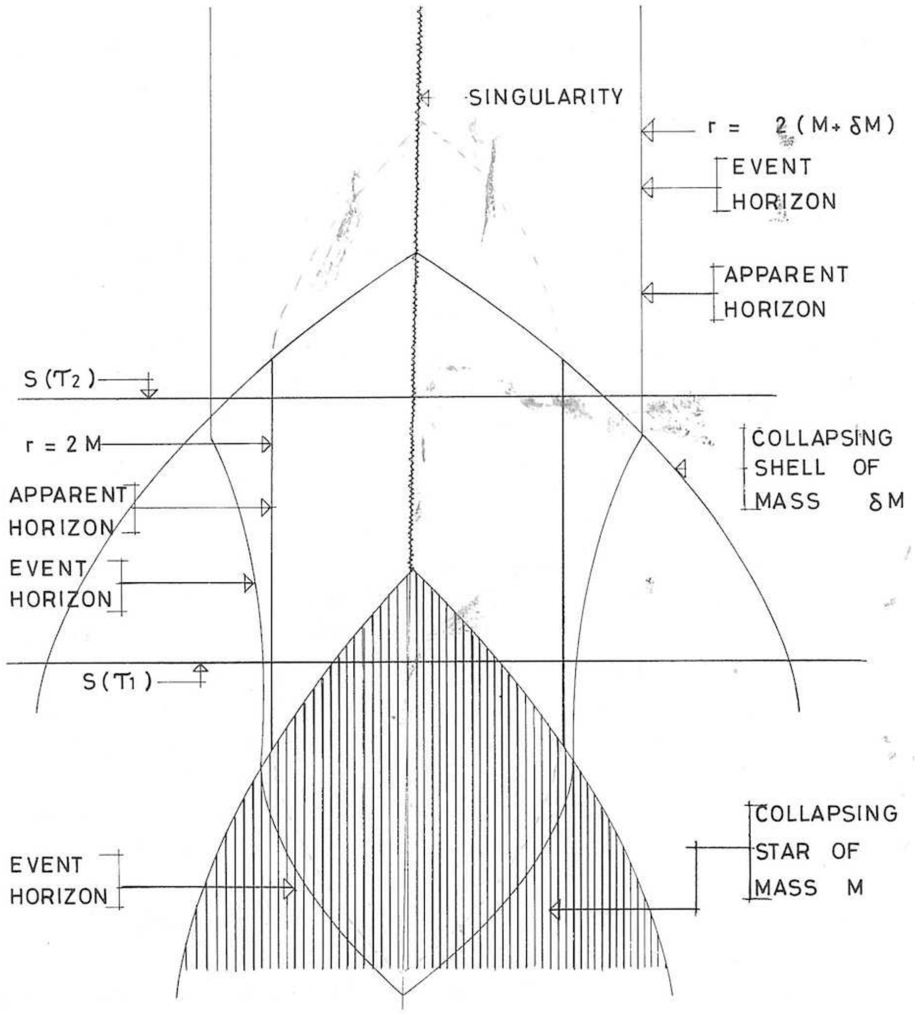
Astrophysical BHs in GR



Emergence of Event Horizon and Trapped Surfaces.

(Hawking, 1972)

Astrophysical BHs in GR



Growth of Event Horizon by a collapsing shell.

(Hawking, 1972)

Objects and ideas

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- Stellar remnants: accreting BH and NS (microquasars, ...)

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- GR effects taken into account: acceleration to relativistic speed • strong gravity lensing, energy shifts • variability time-scales → orbiting near ISCO • gw–elmg. synergy (TDEs)...

GR relevant for BH accretion

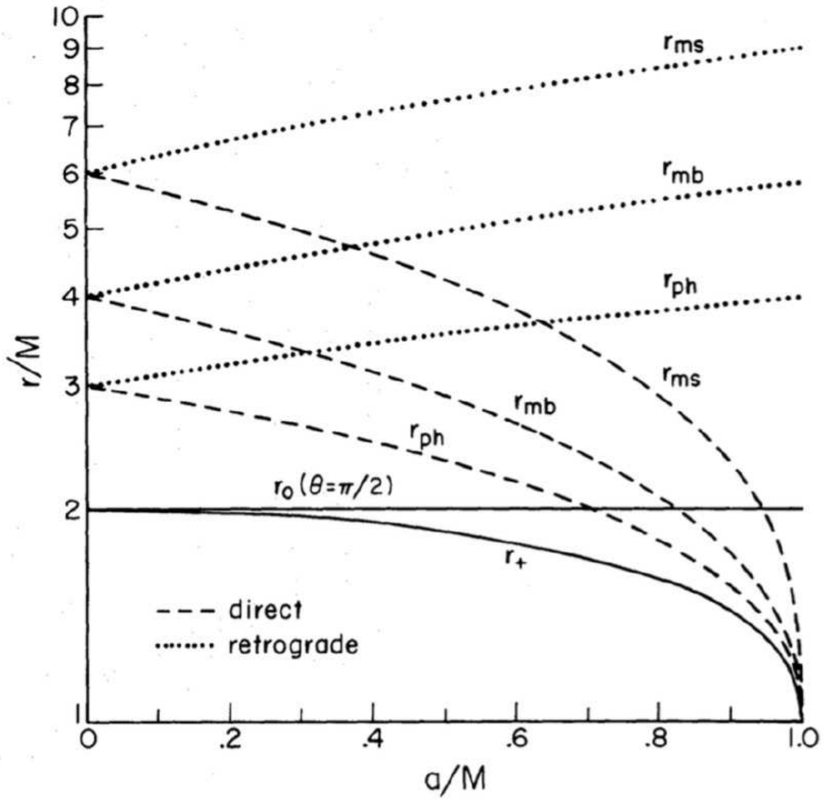


FIG. 1.—Radii of circular, equatorial orbits around a rotating black hole of mass M, as functions of the hole's specific angular momentum a. Dashed and dotted curves (for direct and retrograde orbits) plot the Boyer-Lindquist coordinate radius of the innermost stable (ms), innermost bound (mb), and photon (ph) orbits. Solid curves indicate the event horizon (r_+) and the equatorial boundary of the ergosphere (r_0).

Equatorial orbits of particles near a rotating BH ($|a/M| \leq 1$).

(Bardeen et al, ApJ, 1972)

GR relevant for BH accretion

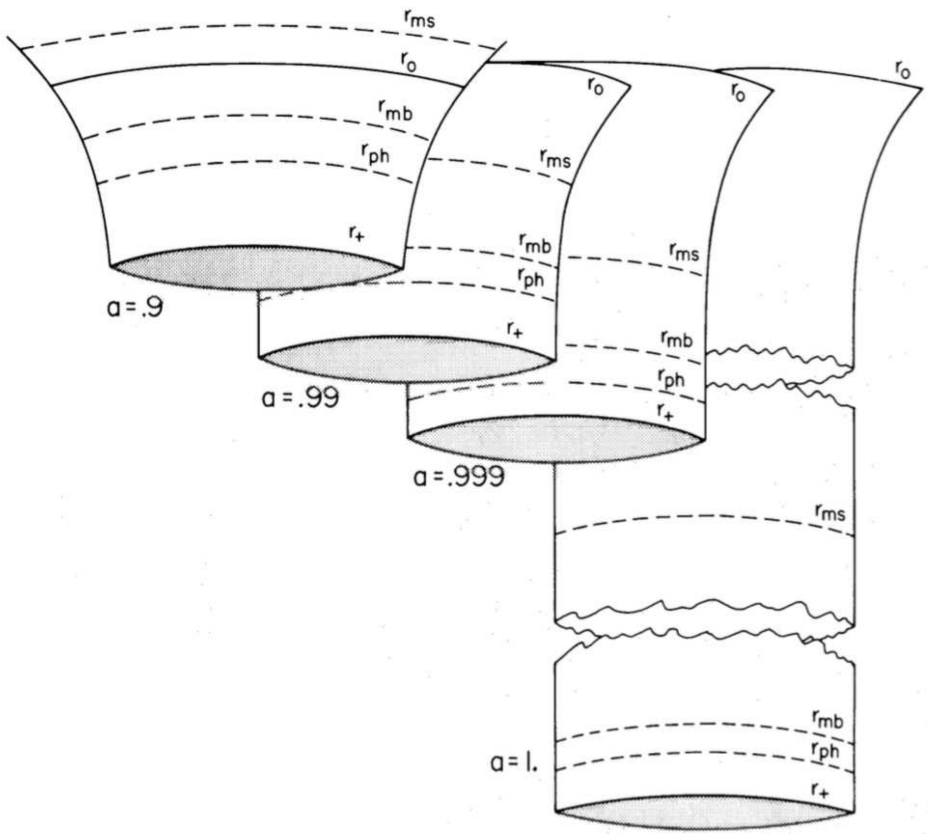
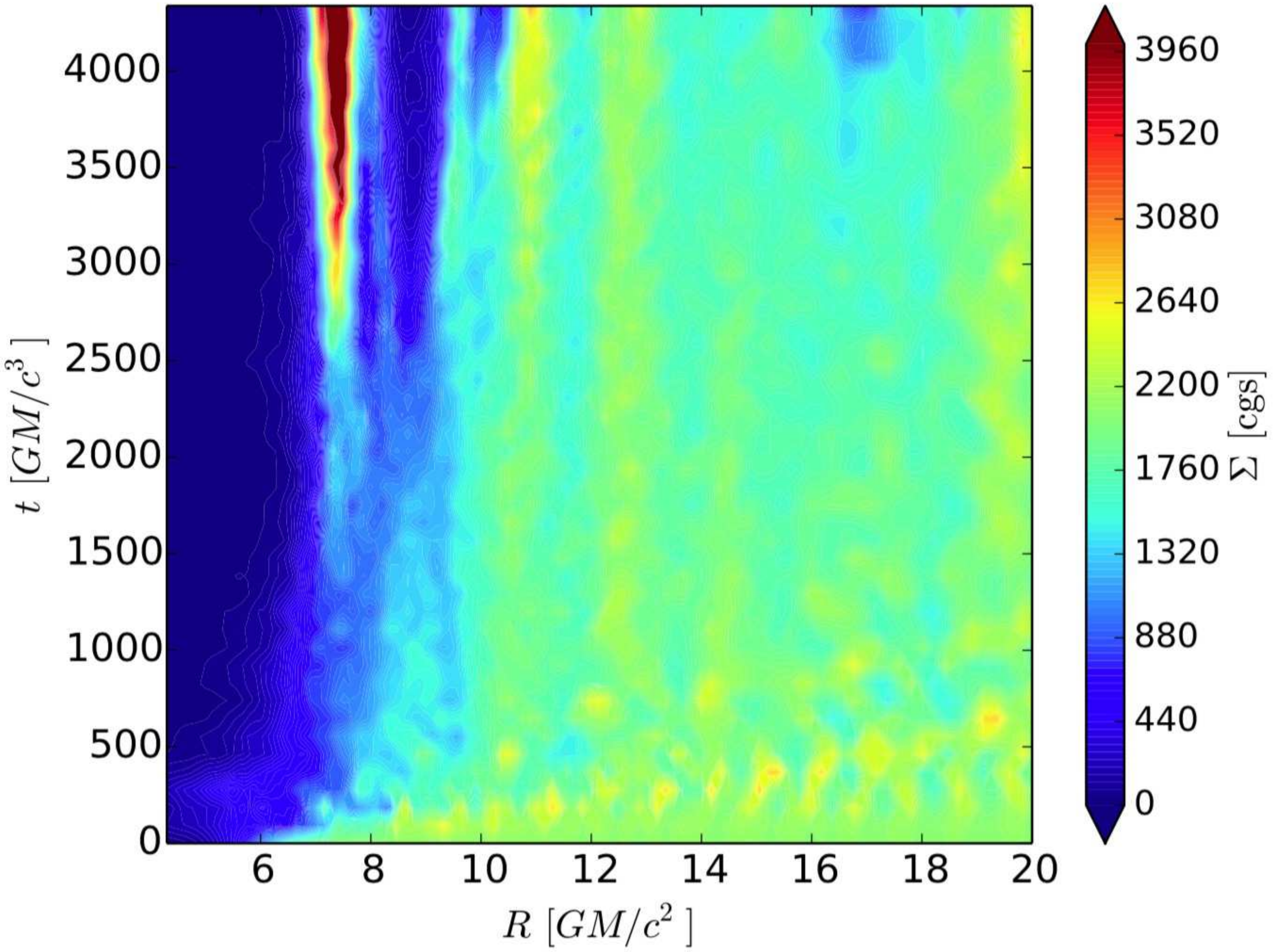


FIG. 2.—Embedding diagrams of the "plane" $\theta = \pi/2$, t = constant, for rotating black holes with near-maximum angular momentum. Here a denotes the hole's angular momentum in units of M. The Boyer-Lindquist radial coordinate r determines only the circumference of the "tube." When $a \to M$, the orbits at r_{ms}, r_{mb}, and r_{ph} all have the same circumference and coordinate radius, although—as the embedding diagram shows clearly—they are in fact distinct.

Embeddings of equatorial plane near a rotating BH ($|a/M| \leq 1$).

(Bardeen et al, ApJ, 1972)

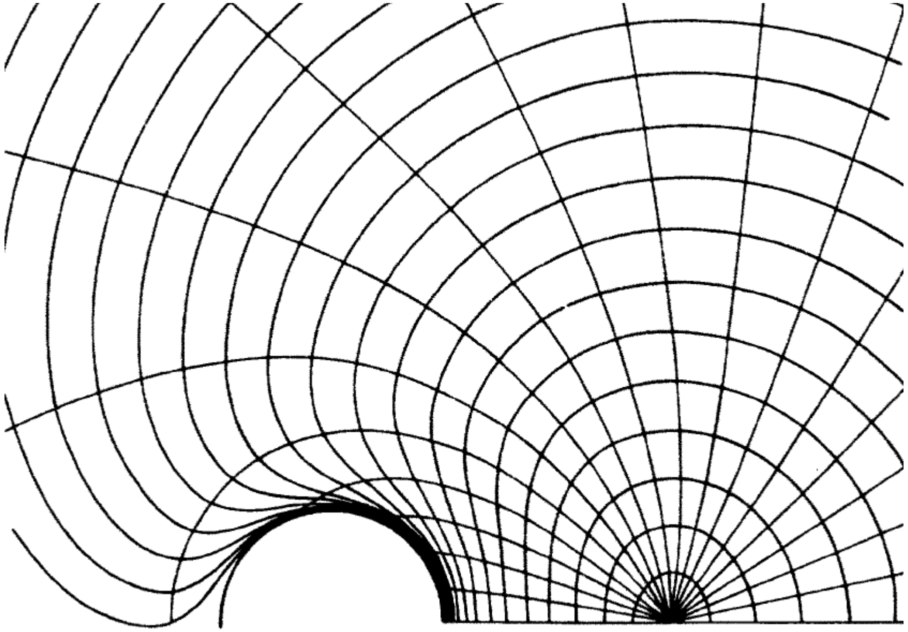
GR relevant for BH accretion



Three-dimensional, global, radiative GRMHD simulations of a thermally unstable disc.

(B. Mishra et al, MNRAS, 2016)

Wave fronts in a BH spacetime



Assumptions and facts: elmg. radiation does not influence geometry of the BH spacetime (to first order).

Wave fronts do not depend on polarization (in geometrical optics approximation).

The analogy:
light propagation in a vacuum curved spacetime versus material media in a flat spacetime.

The effective permeability: $\mu = \sqrt{1 - 2M/r}$.

Mashoon (1973); Hanni (1977); ...

Wave fronts in a BH spacetime

Schwarzschild metric,

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

Eikonal equation,

$$-\left(1 - \frac{2M}{r}\right) (\psi_{,r})^2 + \left(1 - \frac{2M}{r}\right)^{-1} (\psi_{,t})^2 - r^{-2} (\psi_{,\phi})^2 = 0.$$

Solved by separation of variables, $\psi(t, r, \phi) \equiv R(r) + \alpha\phi - \omega t$,

$$\left(1 - \frac{2M}{r}\right) (R')^2 = \left(1 - \frac{2M}{r}\right)^{-1} \omega^2 - r^{-2} \alpha^2.$$

Wave front: $\boxed{\psi(t_0 + n\,\delta t, r, \phi) = \psi(t_0, r_0, 0).}$

Wave fronts in a BH spacetime

Kerr metric,

$$ds^2 = -\frac{\Delta}{\Sigma}\Big(dt - a\sin^2\theta\, d\phi\Big)^2 + \frac{\Sigma}{\Delta}\,dr^2 + \Sigma\, d\theta^2$$

$$+\frac{\sin^2\theta}{\Sigma}\Big[a\,dt - (r^2+a^2)\,d\phi\Big]^2.$$

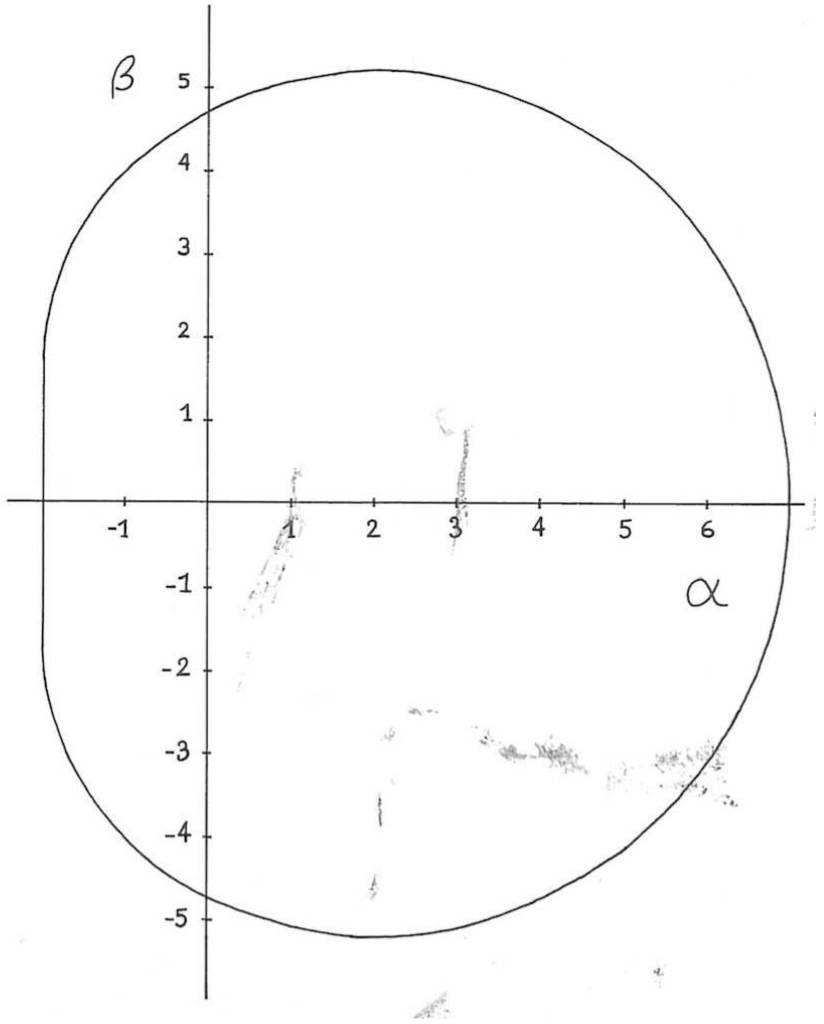
The separation of variables and solution for the eikonal equation follow from Carter's solution of the scalar wave equation,

$$\psi = R(r) + T(\theta) + \alpha\phi - \omega t.$$

Notice: direct vs. retrograde spin.

Wave fronts exhibit the **frame dragging effect.**

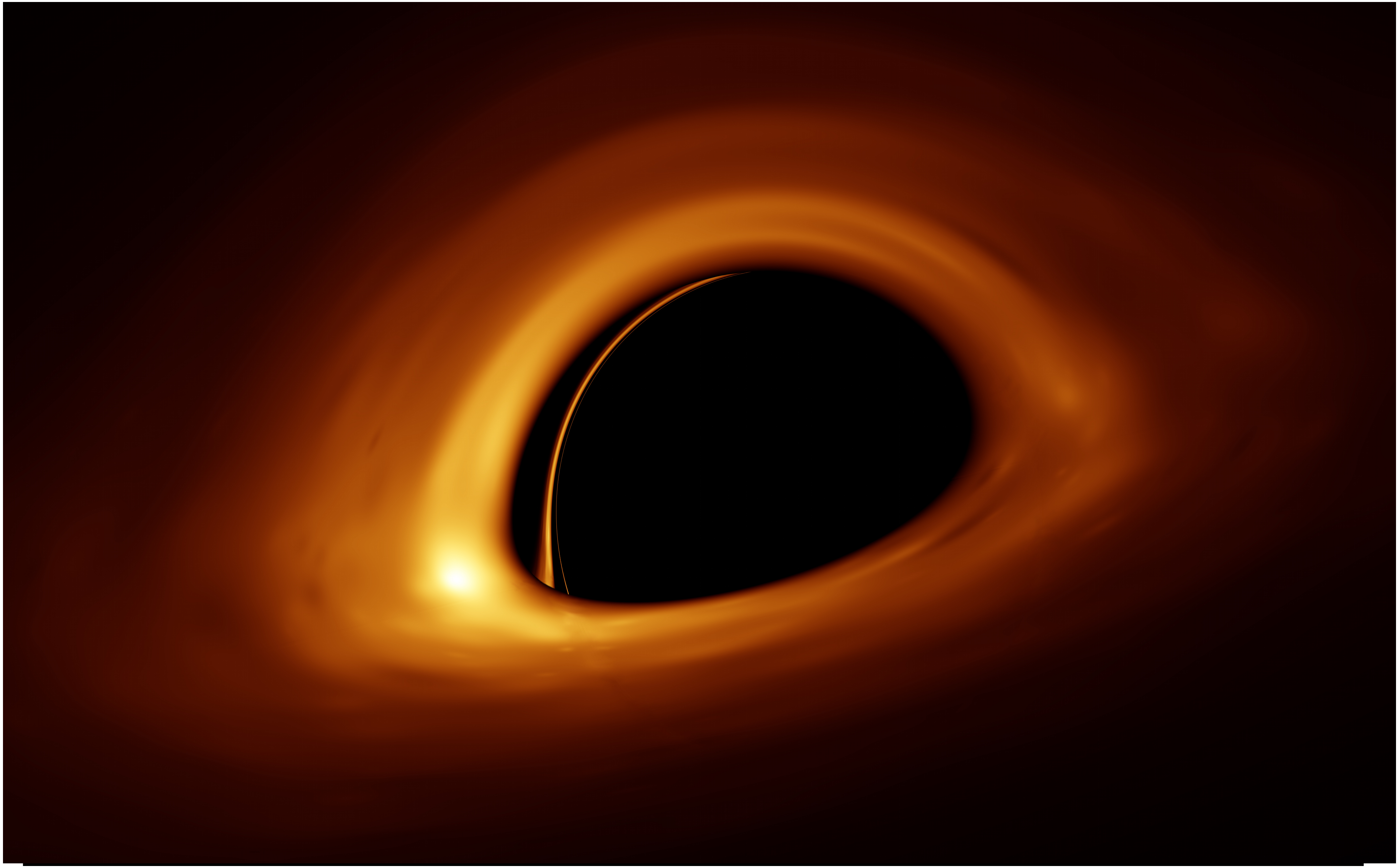
BH shadow and light circle



Apparent shape of an illuminated extreme Kerr BH.

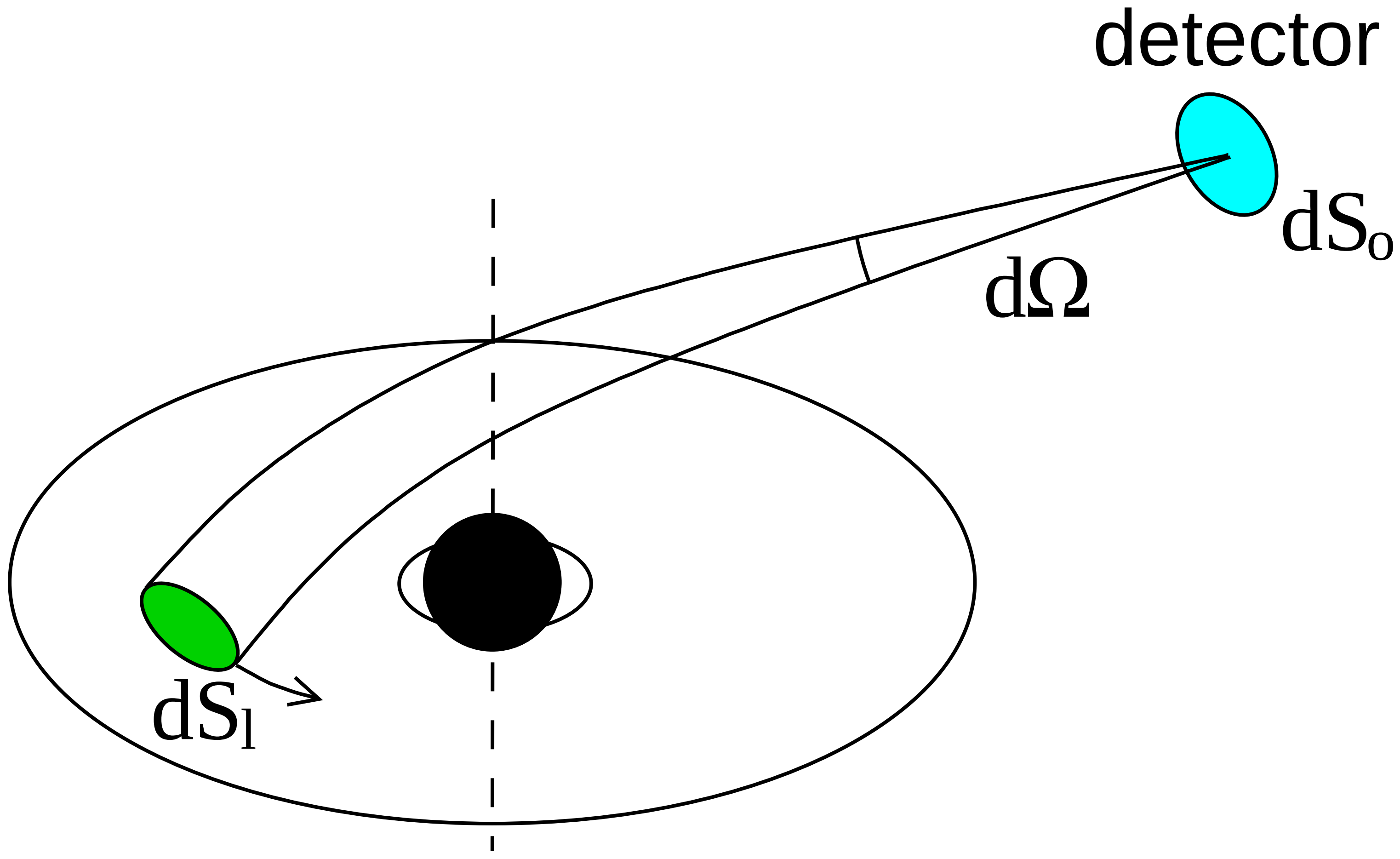
(Bardeen, 1972)

BH shadow and light circle



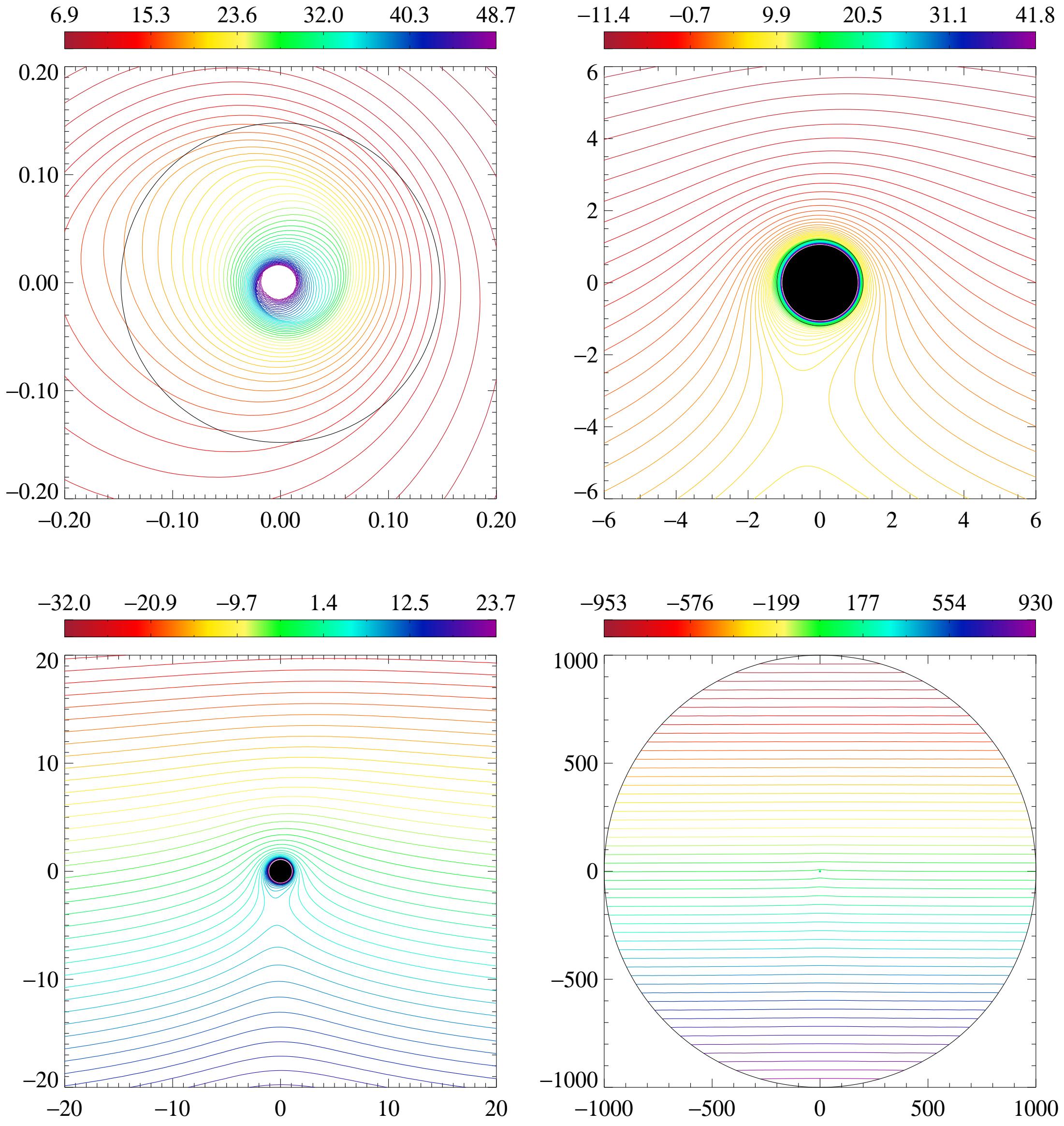
(Bursa et al. 2007)

Light-travel time effect



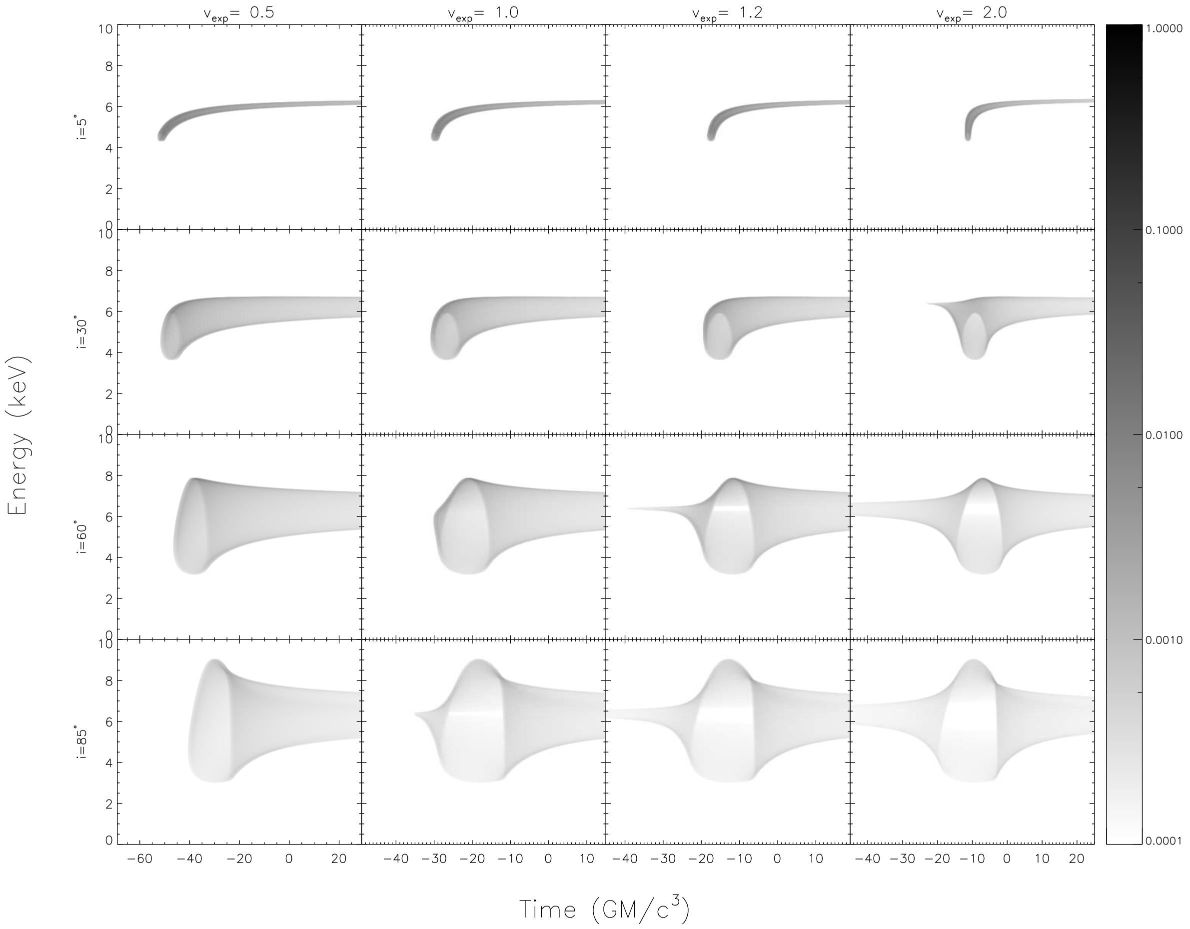
Light-travel time effect

Time delay in BL coord. (a/M=0.9987 , θ_o=70° , r_h=1.05 , r_{ms}=1.198)



Dovčiak et al, 2014

Reverberation from TDEs

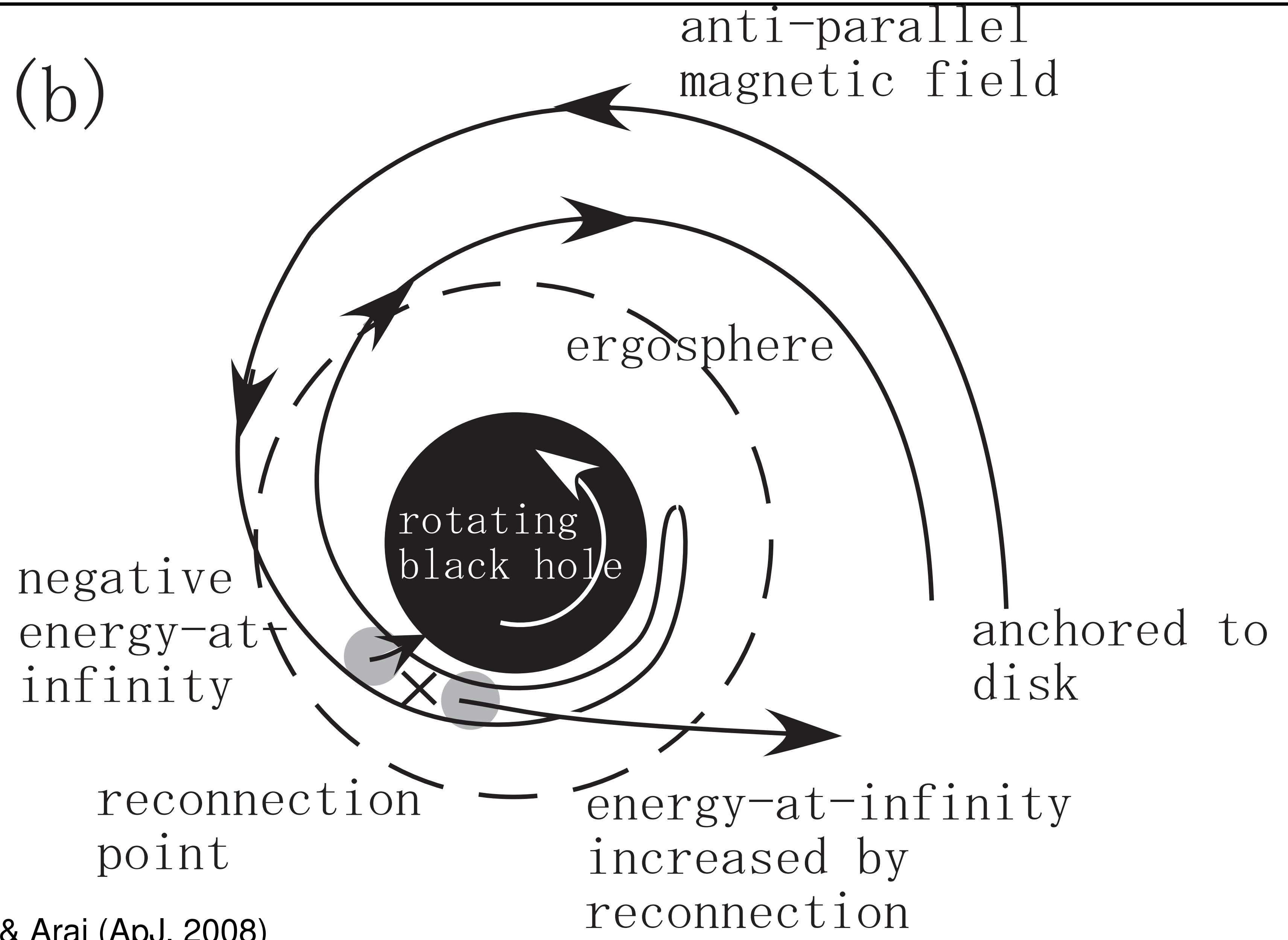


Zhang et al (2015), ApJ

Prospects for future: reverberation signatures
on remnants from tidal disruptions.

(Karas et al, Proc. RAGtime 2014; Kara et al, Nature 2016)

Reconnection in ergosphere?



Koide & Arai (ApJ, 2008)

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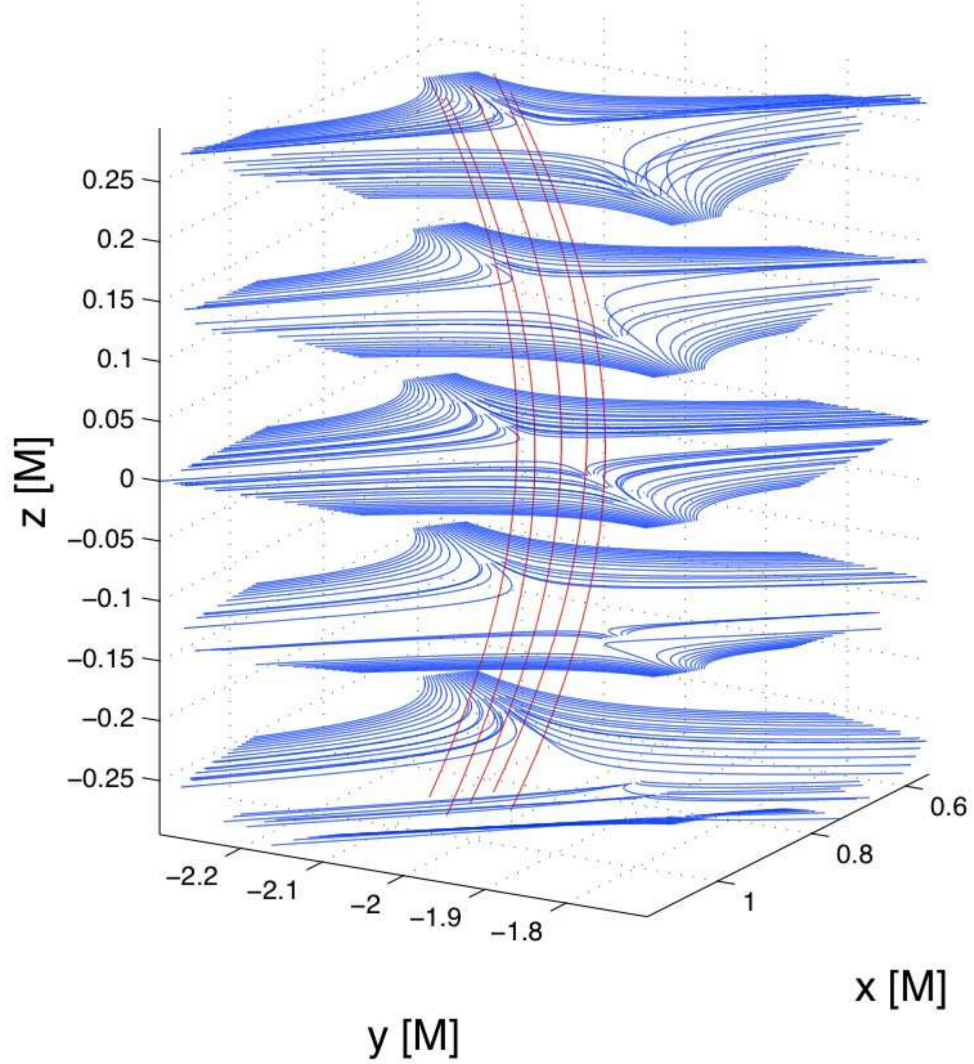
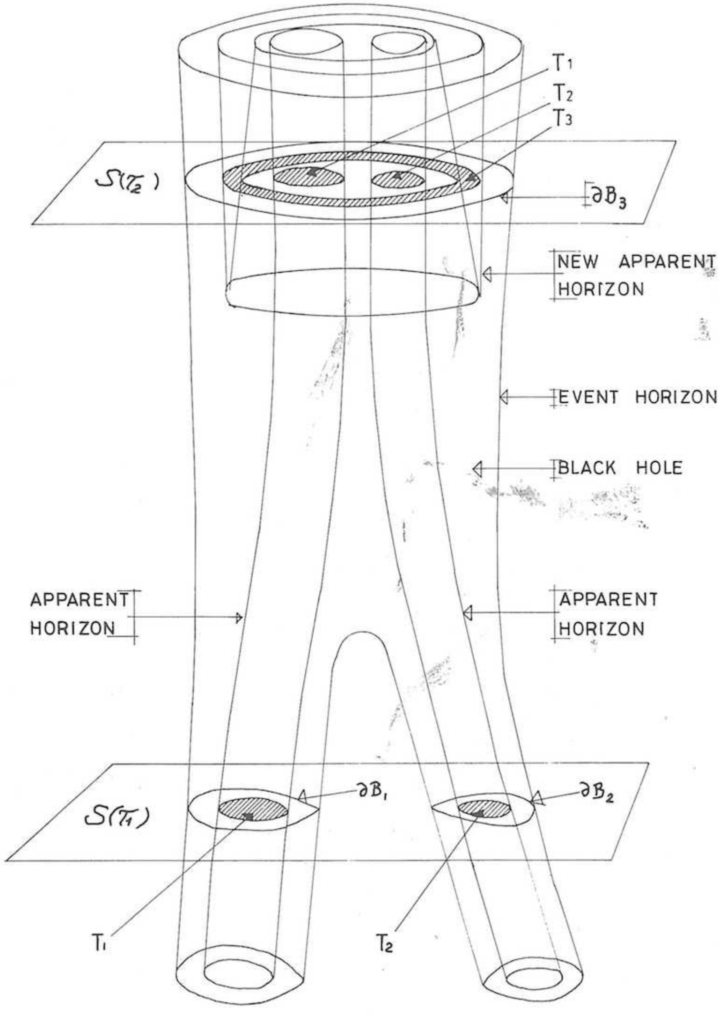


Karas, Kopáček & Kunneriath (CQG, 2012)

Binary BHs

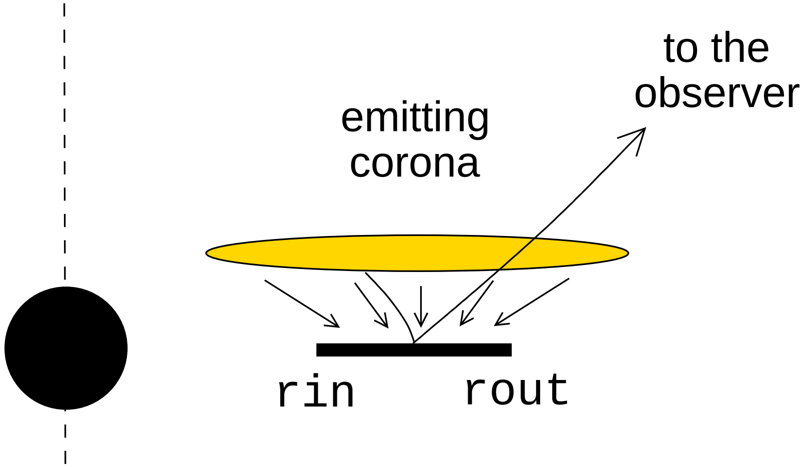


(Hawking, 1972)

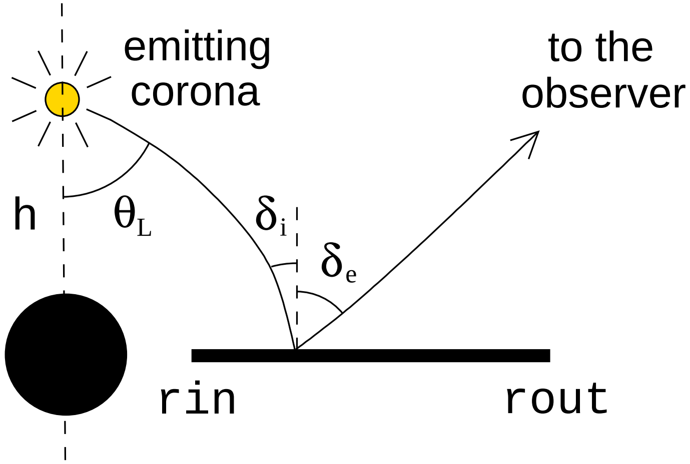
Growth of horizon by BH mergers. Ratio of TDEs/gas accretion. See further lectures at this meeting!

Thank you!

Discussion slides – AD ‘corona’



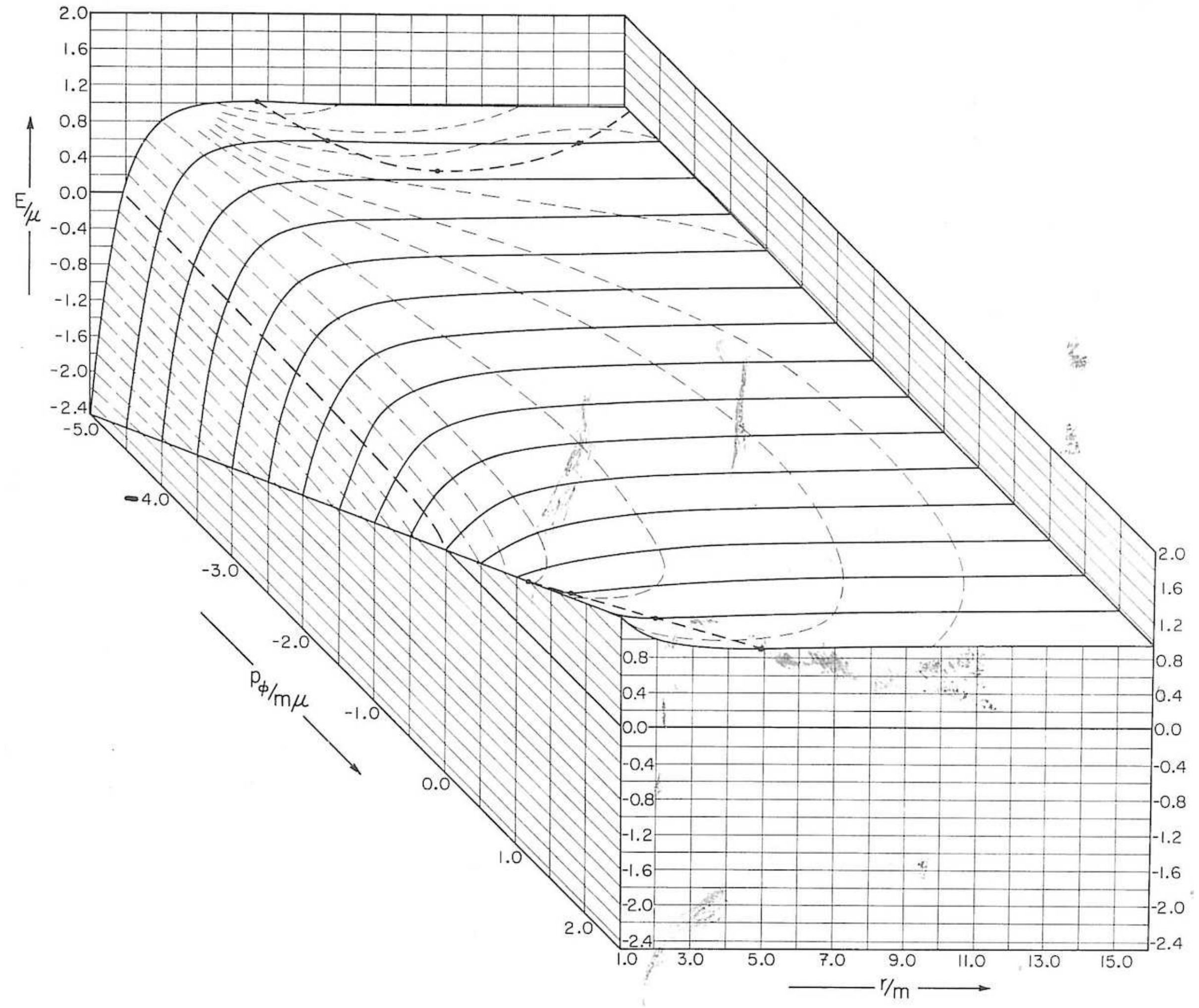
1)



2)

Dovčiak et al, 2004

Discussion slides



Effective potential for test motion near an extreme BH ($a/M = 1$):
co-rotating vs. counter-rotating orbits.

(Ruffini & Wheeler, 1970)