

# Statistical and theoretical studies of flares in Sgr A\*

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Black Hole Accretion and Jets, Oct. 21, 2016@ Kathmandu, Nepal

# OUTLINE

- Flare observations for Sgr A\*
- Statistical study of X-ray flares in Sgr A\*
- Theoretical modeling of flares in Sgr A\*
- Conclusion

# Spectrum and Luminosity of Flares

## ■ Quiescent state

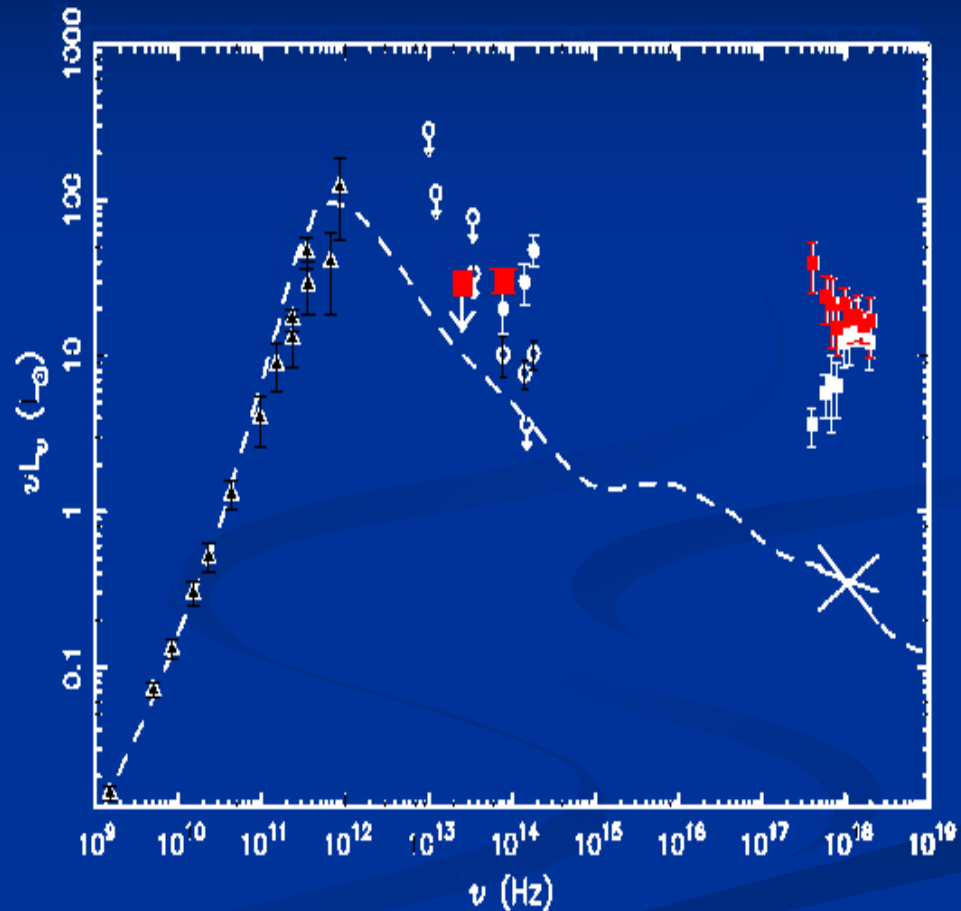
- $L_{\text{bol}} \sim 10^{36}$  erg/s  
 $\sim 10^{-9} L_{\text{Edd}}$  for  $M_{\text{BH}} = 4 \times 10^6 M_{\text{sun}}$  ( $\eta \sim 10^{-6}$ );
- RIAF works (Yuan et al. 2003)

## ■ Flares state

- Mainly at NIR and X-ray, also in radio and submm
- brightest X-ray flares: 400x quiescent luminosity

## ■ Spectrum of flares

- NIR:  $\alpha_{\text{NIR}} \sim 0.4$  ( $\nu L_{\nu} \sim \nu^{\alpha}$ )
- X-ray:  $\alpha_{\text{X}} \sim 0$

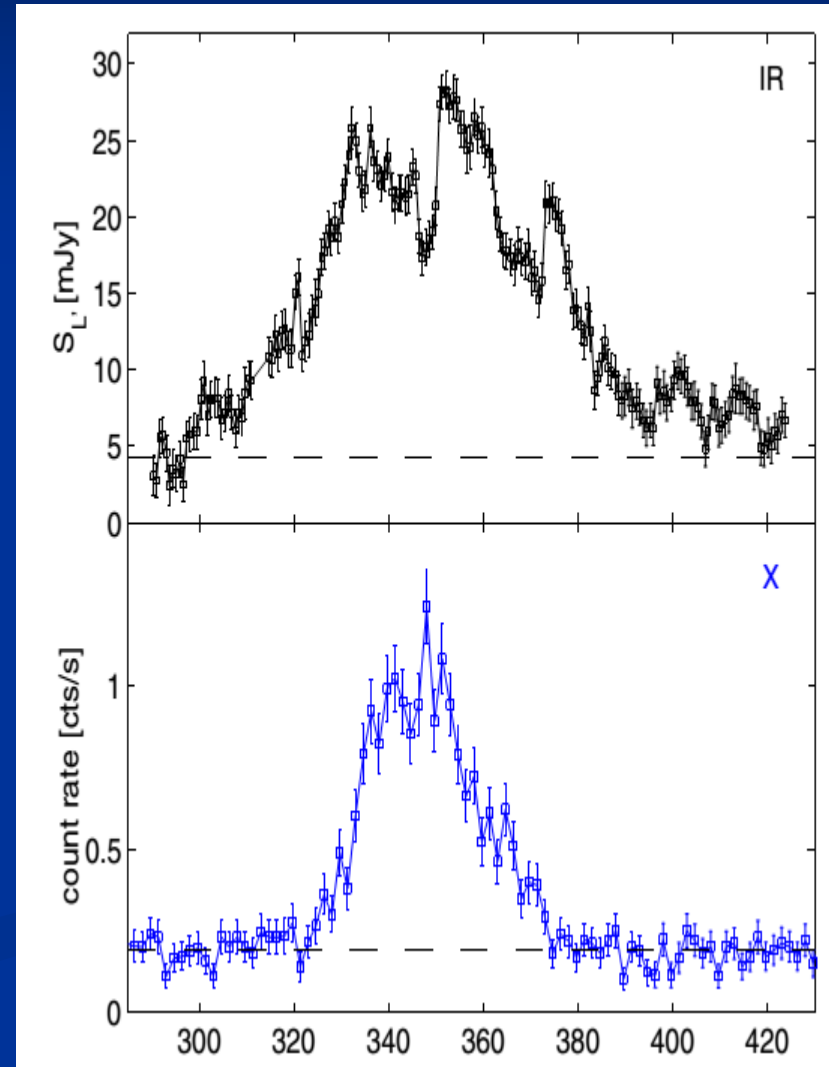


Hornstein et al. 2007; Porquet et al. 2003, 2008;  
Dodds-Eden et al. 2009; Nowark et al. 2012, Neilsen et al. 2013;...

# Light Curves of Flares

Baganoff et al. 2003; Eckart et al. 2006; Hornstein et al. 2007;  
Dodds-Eden et al. 2009; Neilsen et al. 2013; ...

- FWHM of NIR  $\sim$  60 mins, X-ray  $\sim$  30 mins
- substructure in NIR but *not* in X-ray flares?
- X-ray and NIR flares occur simultaneously
- Occurrence rate:
  - IR/NIR :  $\sim$ 4 per day
  - X-ray :  $\sim$ 1 per day
  - every X-ray flare associated with the NIR flare, but *not* vice versa
- Flares also seen in mm, radio, but with smaller amplitudes, broader profiles (e.g., Yusef-Zadeh+06)



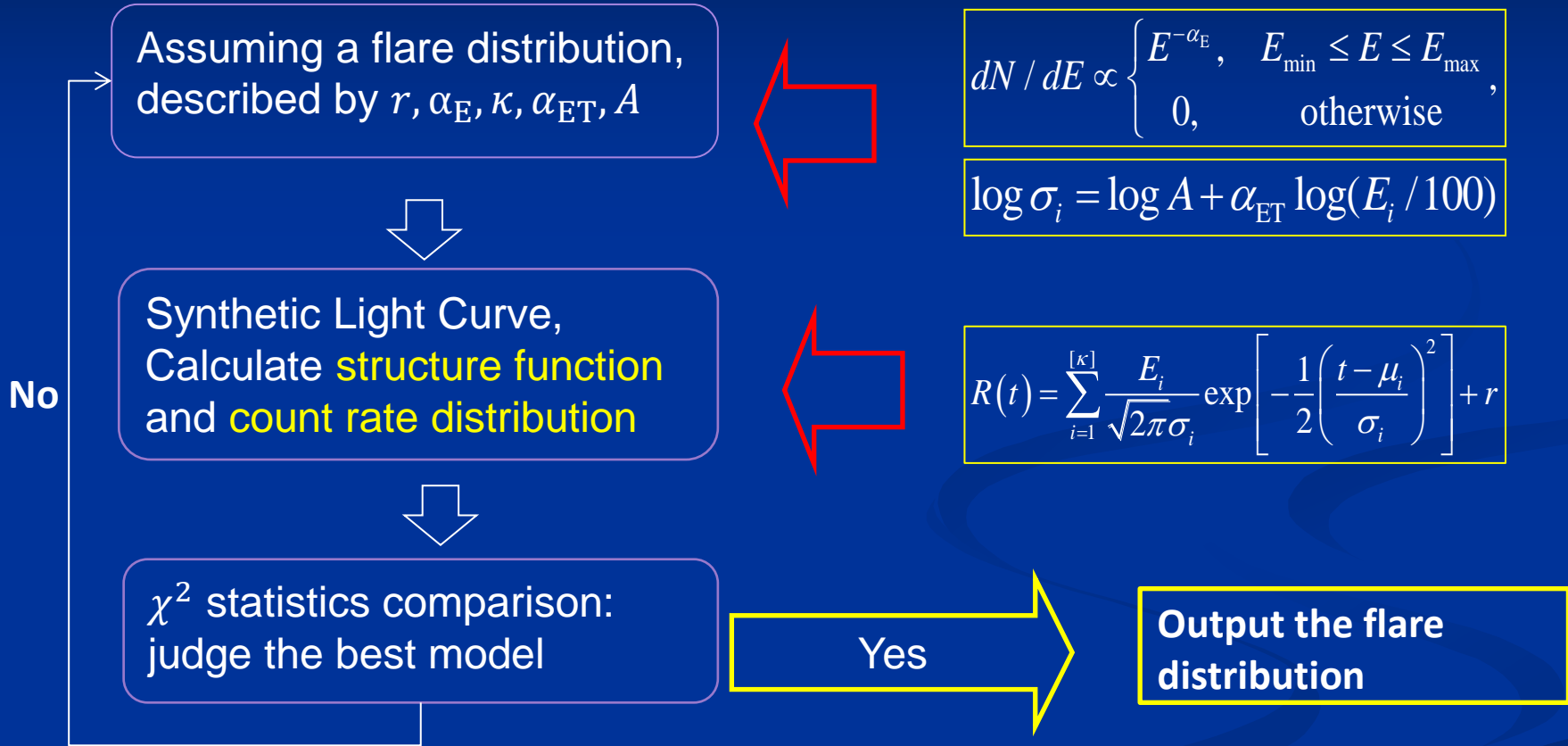
# Proposed models for the flares

- Particle acceleration
  - **magnetic reconnection** (Yuan et al. 2004; Eckart et al. 2006; Dodds-Eden et al. 2009)
  - **shock** (Markoff et al. 2001)
- Dynamical models
  - **Accretion instability** (Tagger & Melia 2006; Falanga et al. 2008)
  - **Orbiting hot spot** (e.g., Broderick & Loeb 2005; Hamaus et al. 2009)
  - **Expanding plasma blob** (Yusef-Zadeh et al. 2006, Eckart et al. 2006, Dodds-Eden et al. 2010; Kusunose & Takahara 2011; Trap et al. 2011)
  - **Tidal disruption of asteroids** (Cadez et al. 2008; Kostic et al. 2009; Zubovas et al. 2012)

Almost all of these works are **phenomenological**.

**Statistical study of the X-ray  
flares in Sgr A\***

# Methodology



$$dN / dE \propto \begin{cases} E^{-\alpha_E}, & E_{\min} \leq E \leq E_{\max} \\ 0, & \text{otherwise} \end{cases}$$

$$\log \sigma_i = \log A + \alpha_{ET} \log(E_i / 100)$$

$$R(t) = \sum_{i=1}^{[\kappa]} \frac{E_i}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{1}{2}\left(\frac{t - \mu_i}{\sigma_i}\right)^2\right] + r$$

$$\chi^2 = \chi_{CR}^2 + \chi_{SF}^2$$

$$\chi_{SF}^2 = (\log \mathbf{V}_{\text{obs}} - \log \mathbf{V}_{\text{sim}})^T \mathbf{C}_{SF}^{-1} (\log \mathbf{V}_{\text{obs}} - \log \mathbf{V}_{\text{sim}})$$

$$\chi_{CR}^2 = (\mathbf{n}_{\text{obs}} - \mathbf{n}_{\text{sim}})^T \mathbf{C}^{-1} (\mathbf{n}_{\text{obs}} - \mathbf{n}_{\text{sim}})$$

# MCMC fitting:

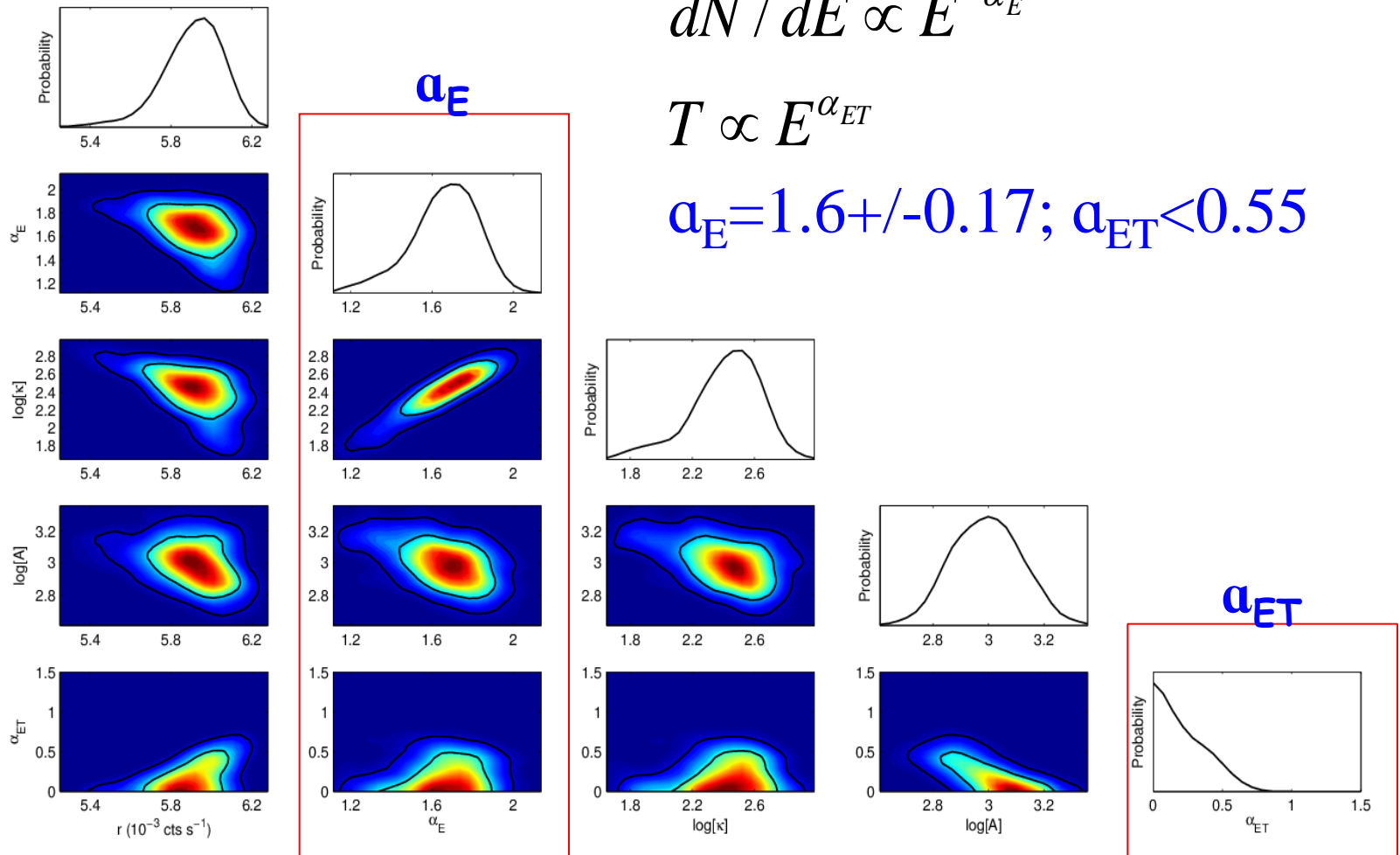
Power-law distributions for flare fluence and fluence-duration correlation

Li et al. 2015b

$$dN / dE \propto E^{-\alpha_E}$$

$$T \propto E^{\alpha_{ET}}$$

$$\alpha_E = 1.6 \pm 0.17; \alpha_{ET} < 0.55$$





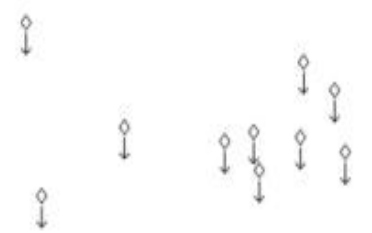
# Self-Organized Criticality (SOC): statistical approach

The prototype of SOC system



SOC:

A class of dynamical system with nonlinear energy dissipation that is slowly and continuously driven toward a critical value of an instability threshold.



SUB-CRITICAL STATE :  
NO AVALANCHES



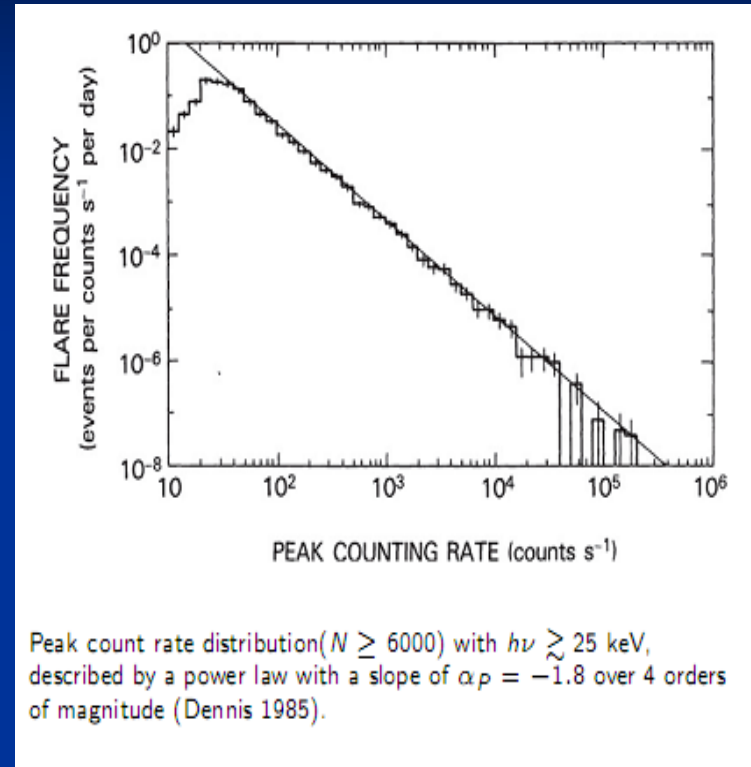
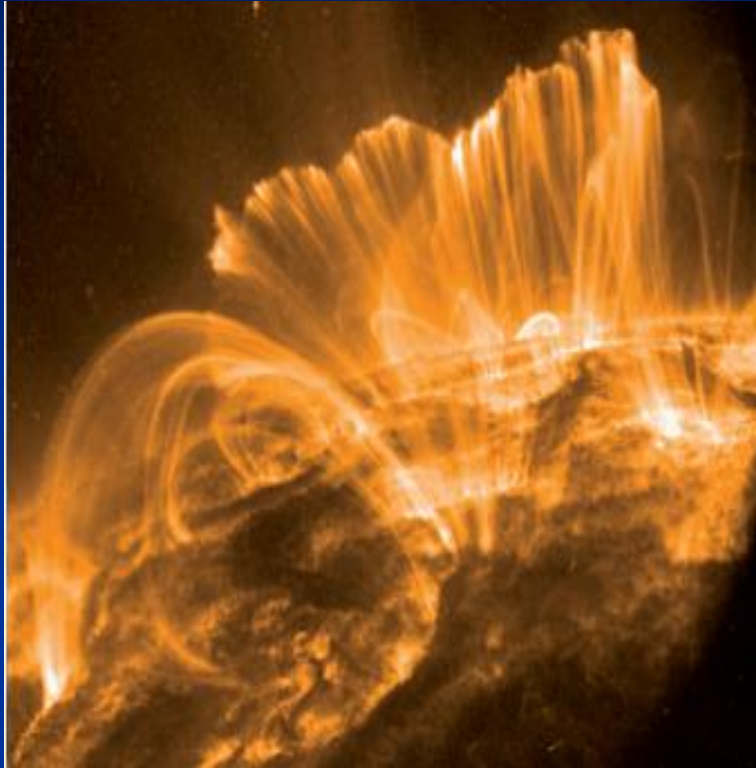
CRITICAL STATE :  
AVALANCHES

Consequences:

Power-law distributions of various event parameters, such as peak energy, total energy (fluence) and durations.

# Solar flares:

## Statistics results and Self-Organized Criticality (SOC)



- power-law distributions also observed in flare duration, total energy and flux.
- $\mathcal{Q} \rightarrow \mathcal{K}$ , consistent with an  $S=3$  SOC system driven by magnetic reconnection in the solar atmosphere.
- power-law index can be used to diagnose the dimensionality ( $S$ ) of the process leading to flares, and further their physical mechanism;

# Confrontation with SOC theory

## Simulation Results:

- power-law distributions of fluence and width-fluence correlation;

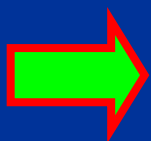
## Fractal-diffusive SOC theory:

- power-law indexes depend on the dimension  $S$ ;
- For  $S=3$  ( $\beta \sim 1.0$ ,  $D_S \sim (S+1)/2$ )

BEST-FIT MODEL PARAMETERS WITH THE MCMC METHOD

$r$ ( $10^{-3}$ cts $s^{-1}$ )	$\alpha_E$	$\log(\kappa)$	$\log(A)$	$\alpha_{ET}$	$\chi^2_\nu$
$5.90 \pm 0.14$	$1.65 \pm 0.17$	$2.41 \pm 0.22$	$2.99 \pm 0.12$	$< 0.55$	0.9

$$\alpha_E^{\text{th}} = 1 + \frac{S-1}{D_S + 2/\beta} \approx 1.5$$
$$\alpha_{ET}^{\text{th}} = \frac{2}{D_S\beta + 2} \approx 0.5$$



$S=3$  for X-ray flares from Sgr A\*

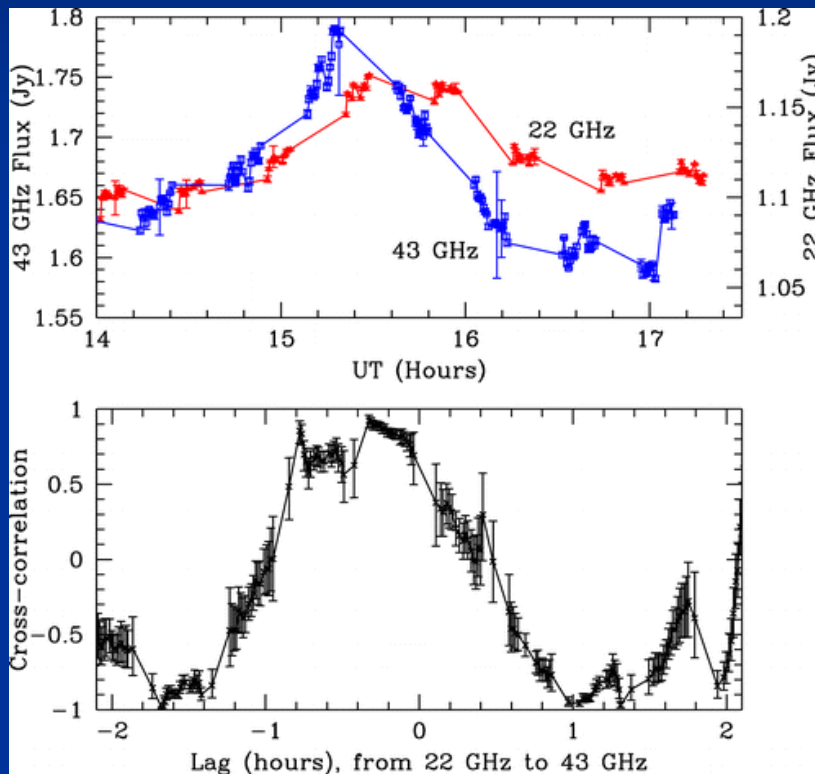
# Implications on the nature of X-ray flares of Sgr A\*

- Statistical properties of Sgr A\* are well consistent with  $S=3$  fractal-diffusive SOC theory, same as solar flares
- Two Implications:
  - Flares in Sgr A\* originate from reconnection
  - in the surface of accretion flow, not in the jet
- This is also supported by:
  - Flares are associated ejection of blobs (Yusef-Zadeh et al. 2006), which is caused by reconnection (second part of this talk)

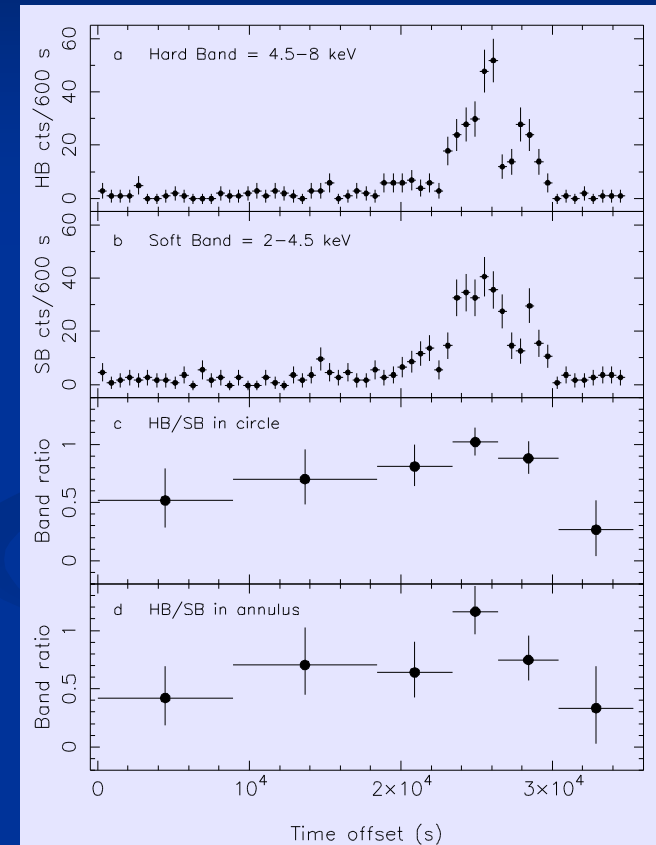
# Theoretical model for the flares in Sgr A\*

# Time lag as evidence for episodic ejection of radio blobs

Yusef-Zadeh+2006,2008; Brinkerink+2015;.....



Radio light curves and cross correlation



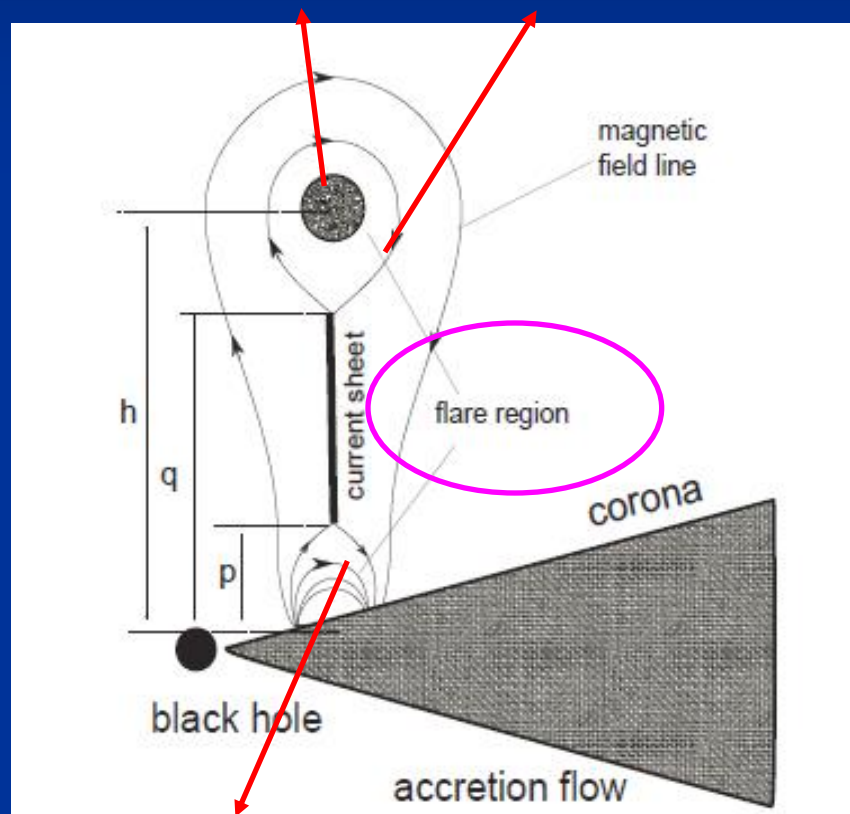
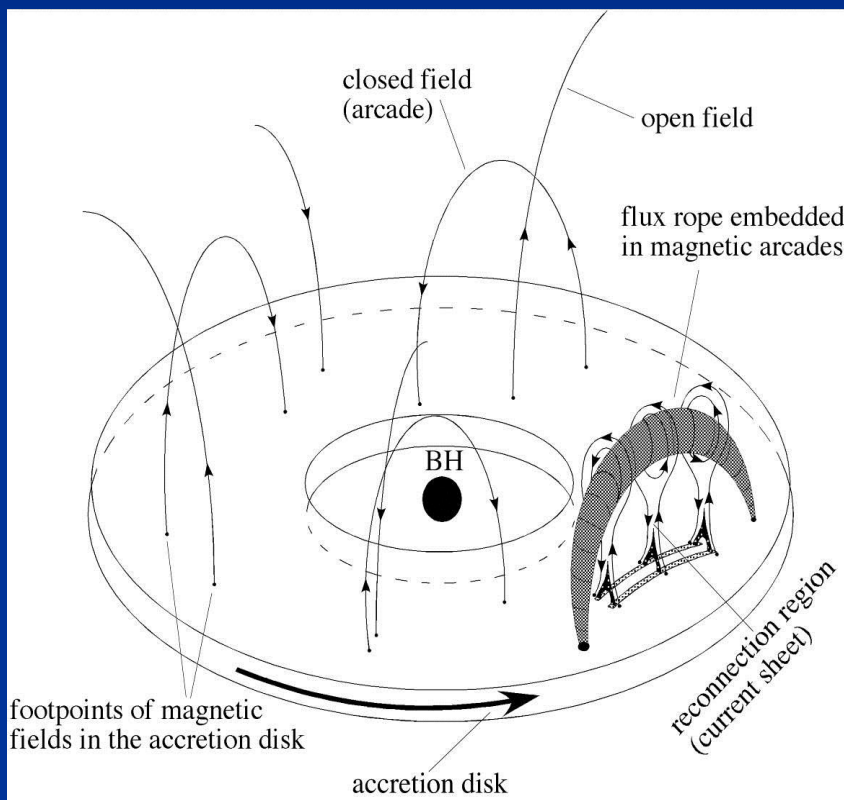
X-ray flare

associated with X-ray, IR & radio flares

# Model for flare & ejection of blobs

Yuan et al. 2009

- a) radio & submm flares? also IR & X-ray?
- b) Time lags between different wavelengths



By analogy with coronal mass ejections (CMEs)/solar flares

synchrotron of electrons accelerated by reconnection:  
IR & X-ray flares?

# Equations for Dynamical Evolutions of Blobs

$$\gamma_b \frac{d^2 h}{dt^2} = \frac{B_0^2 \lambda^4}{8hmL_{PQ}^2} \left[ \frac{H_{PQ}^2}{2h^2} - \frac{(p^2 + \lambda^2)(h^2 - q^2)}{h^2 + \lambda^2} - \frac{(q^2 + \lambda^2)(h^2 - p^2)}{h^2 + \lambda^2} \right] - \frac{GM_\bullet \gamma_b}{(R_0 + h)^2}$$

$$\frac{dm}{dt} = B_0 M_A \sqrt{\frac{n_e m_H}{\pi}} \frac{\lambda^2 (q - p)(h^2 + \lambda^2)}{(h^2 - y_0^2)(y_0^2 + \lambda^2)} \times \sqrt{\frac{f(y_0)(q^2 - y_0^2)(y_0^2 - p^2)}{(p^2 + \lambda^2)(q^2 + \lambda^2)}}$$

$$\frac{dp}{dt} = p' \dot{h},$$

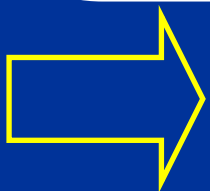
$$\frac{dq}{dt} = q' \dot{h}.$$

with

$$p' = \frac{\tilde{A}_{0h} A_{Rq} - A_{Rh} A_{0q}}{A_{Rp} A_{0q} - A_{0p} A_{Rq}},$$

$$q' = \frac{A_{Rh} A_{0p} - \tilde{A}_{0h} A_{Rp}}{A_{Rp} A_{0q} - A_{0p} A_{Rq}},$$

$$\tilde{A}_{0h} = \frac{cE_z}{B_0 \lambda \dot{h}} + A_{0h} = \frac{M_A V_A B_y(0, y_0)}{B_0 \lambda \dot{h}} + A_{0h},$$



Determining the evolution profiles of  $p$ ,  $q$ ,  $h$ ,  $m$



# Equations Cont'd

$$\begin{aligned}
 A_R &= \frac{\lambda H_{PQ}}{2hL_{PQ}} \ln \left[ \frac{\lambda H_{PQ}^3}{r_{00}L_{PQ}(h^4 - p^2q^2)} \right] \\
 &+ \tan^{-1} \left( \frac{\lambda}{h} \sqrt{\frac{p^2 + \lambda^2}{q^2 + \lambda^2}} \sqrt{\frac{h^2 - q^2}{h^2 - p^2}} \right) \\
 &+ \frac{\lambda}{qL_{PQ}} \left\{ (h^2 - q^2)F \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p}{q} \right] \right. \\
 &+ (q^2 - p^2)\Pi \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p^2 + \lambda^2}{q^2 + \lambda^2}, \frac{p}{q} \right] \\
 &\left. - \frac{H_{PQ}^2}{h^2} \Pi \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p^2}{h^2}, \frac{p}{q} \right] \right\} \\
 &= \frac{\pi}{4} + \ln \left( \frac{2\lambda}{r_{00}} \right),
 \end{aligned}$$

$$\begin{aligned}
 A_{Rp} &= \frac{\lambda p(h^2 + \lambda^2)}{q(p^2 + \lambda^2)^2} \sqrt{\frac{p^2 + \lambda^2}{q^2 + \lambda^2}} \left\langle \left( 1 - \frac{q^2}{h^2} \right) \right. \\
 &\times \Pi \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p^2}{h^2}, \frac{p}{q} \right] \\
 &- F \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p}{q} \right] \\
 &\left. - \frac{q}{2h} \sqrt{\frac{h^2 - q^2}{h^2 - p^2}} \left\{ 1 + \ln \left[ \frac{\lambda H_{PQ}^3}{r_{00}L_{PQ}(h^4 - p^2q^2)} \right] \right\} \right\rangle,
 \end{aligned}$$

$$\begin{aligned}
 A_{Rq} &= \frac{\lambda(h^2 + \lambda^2)}{(q^2 + \lambda^2)^2} \sqrt{\frac{q^2 + \lambda^2}{p^2 + \lambda^2}} \left\langle \left( 1 - \frac{p^2}{h^2} \right) \right. \\
 &\times \Pi \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p^2}{h^2}, \frac{p}{q} \right] \\
 &- F \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p}{q} \right] \\
 &\left. - \frac{q}{2h} \sqrt{\frac{h^2 - p^2}{h^2 - q^2}} \left\{ 1 + \ln \left[ \frac{\lambda H_{PQ}^3}{r_{00}L_{PQ}(h^4 - p^2q^2)} \right] \right\} \right\rangle,
 \end{aligned}$$

$$\begin{aligned}
 A_{Rh} &= \frac{\lambda}{2h^2L_{PQ}H_{PQ}} \left\{ 2 \frac{h^6 - \lambda^2p^2q^2}{h^2 + \lambda^2} \right. \\
 &\left. - \frac{h^2(p^2 + q^2)(h^2 - \lambda^2)}{h^2 + \lambda^2} \right.
 \end{aligned}$$

$$\begin{aligned}
 &\left. + (h^4 - p^2q^2) \ln \left[ \frac{\lambda H_{PQ}^3}{r_{00}L_{PQ}(h^4 - p^2q^2)} \right] \right\} \\
 &+ \frac{\lambda}{hqL_{PQ}} \left\{ (h^2 + q^2)F \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p}{q} \right] \right. \\
 &- q^2E \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p}{q} \right] \\
 &\left. - \frac{h^4 - p^2q^2}{h^2} \Pi \left[ \sin^{-1} \left( \frac{q}{h} \right), \frac{p^2}{h^2}, \frac{p}{q} \right] \right\}.
 \end{aligned}$$

$$\begin{aligned}
 A_0^0 &= \frac{2I_0}{c} \frac{\lambda}{qL_{PQ}} \left[ (h^2 - q^2)K \left( \frac{p}{q} \right) + (q^2 - p^2) \right. \\
 &\left. \times \Pi \left( \frac{p^2 + \lambda^2}{q^2 + \lambda^2}, \frac{p}{q} \right) - \frac{H_{PQ}^2}{h^2} \Pi \left( \frac{p^2}{h^2}, \frac{p}{q} \right) \right],
 \end{aligned}$$

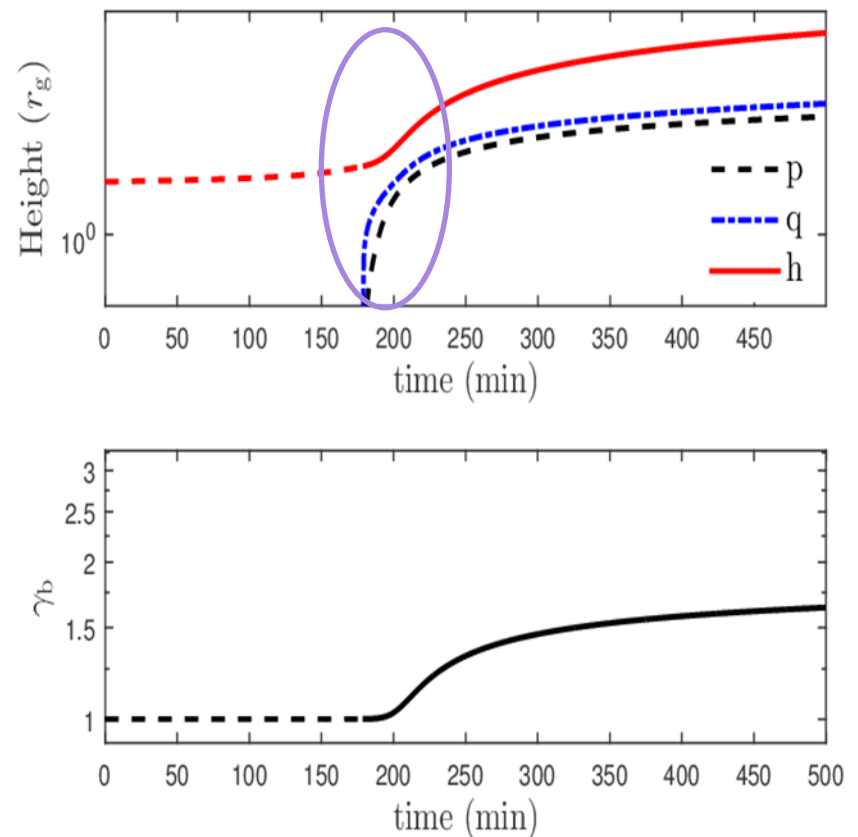
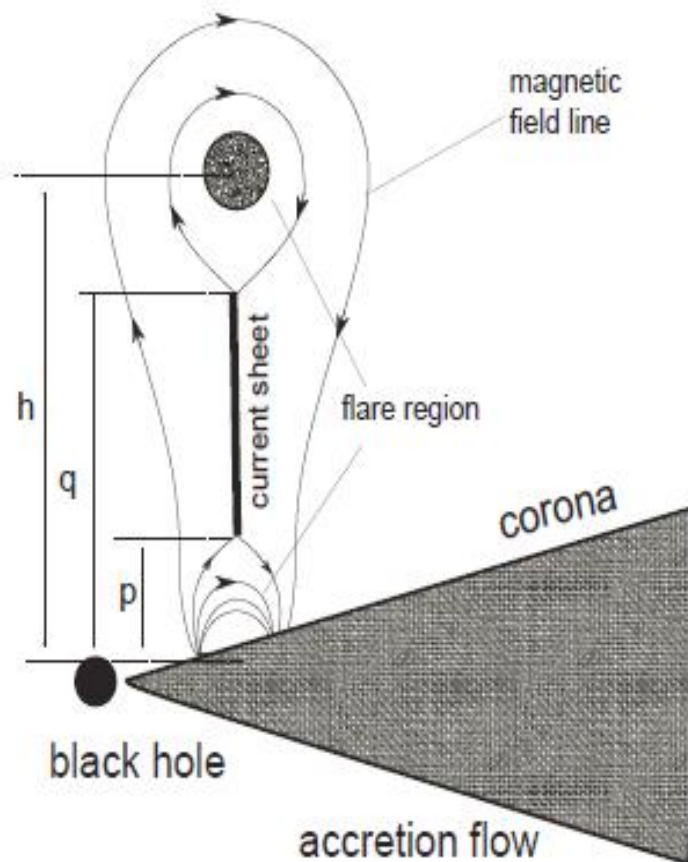
$$\begin{aligned}
 A_{0p} &= \frac{\lambda p(h^2 + \lambda^2)(q^2 + \lambda^2)}{h^2q[(p^2 + \lambda^2)(q^2 + \lambda^2)]^{3/2}} \\
 &\times \left[ (h^2 - p^2)\Pi \left( \frac{p^2}{h^2}, \frac{p}{q} \right) - h^2K \left( \frac{p}{q} \right) \right],
 \end{aligned}$$

$$\begin{aligned}
 A_{0q} &= \frac{\lambda(h^2 + \lambda^2)(p^2 + \lambda^2)}{h^2[(p^2 + \lambda^2)(q^2 + \lambda^2)]^{3/2}} \\
 &\times \left[ (h^2 - q^2)\Pi \left( \frac{p^2}{h^2}, \frac{p}{q} \right) - h^2K \left( \frac{p}{q} \right) \right],
 \end{aligned}$$

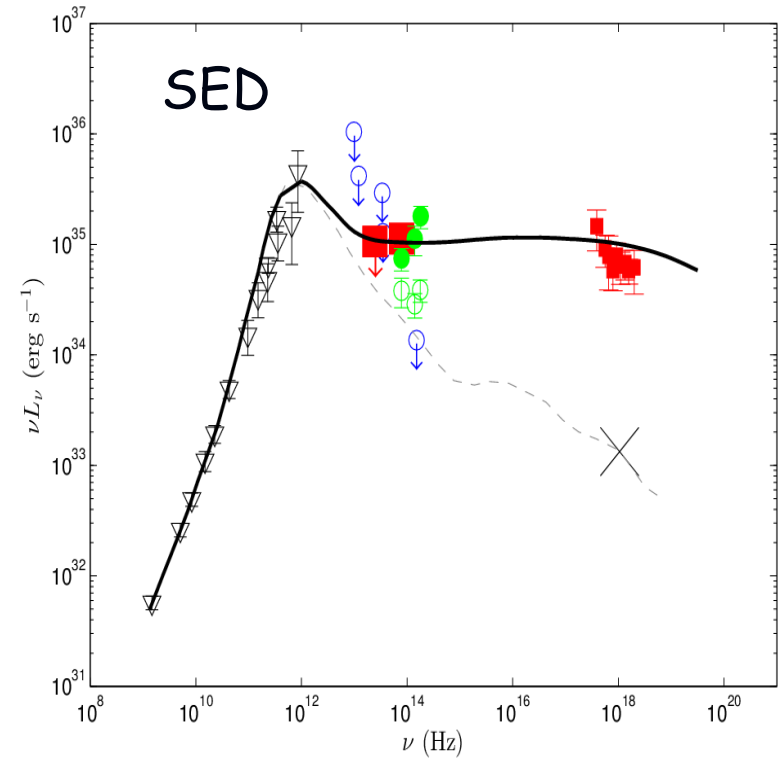
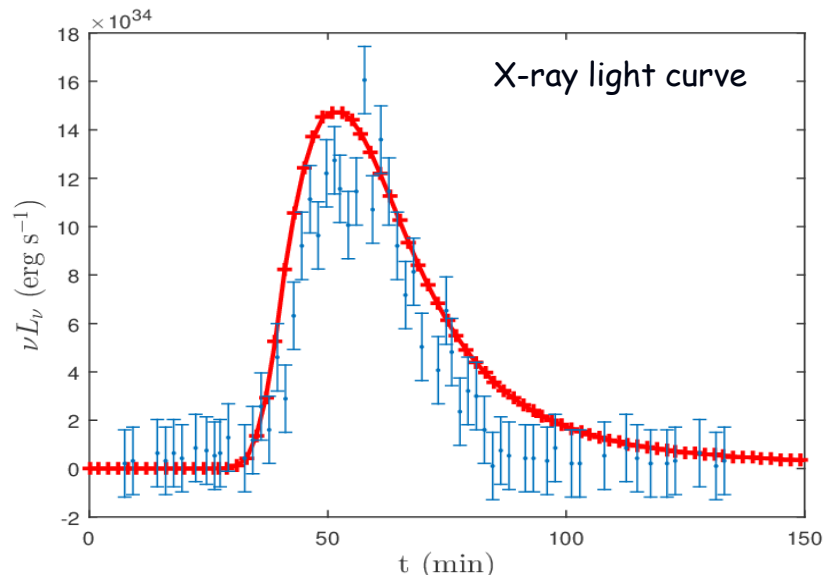
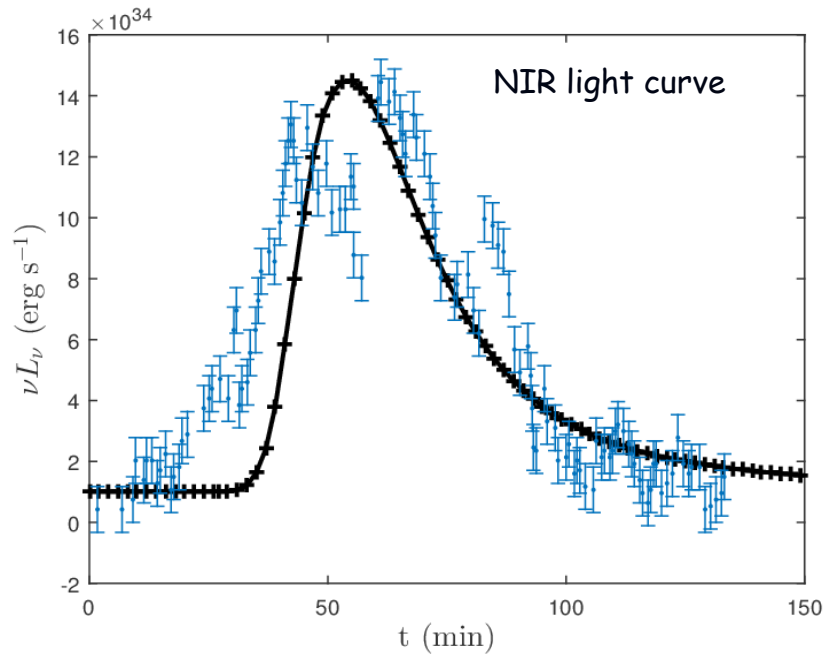
$$\begin{aligned}
 A_{0h} &= -\frac{\lambda}{h^3q\sqrt{(p^2 + \lambda^2)(q^2 + \lambda^2)}} \\
 &\times \left[ h^2q^2E \left( \frac{p}{q} \right) - h^2(h^2 + q^2)K \left( \frac{p}{q} \right) \right. \\
 &\left. + (h^4 - p^2q^2)\Pi \left( \frac{p^2}{h^2}, \frac{p}{q} \right) \right].
 \end{aligned}$$

# Catastrophic dynamical evolution of the current sheet and flux rope

Li et al. 2016, in preparation



# NIR & X-ray Light curves and SED



Main features:

1. Simultaneous flaring
2. Quasi-symmetric profile
3. Amplitudes both in NIR and X-ray

# Summary

- Statistics of flares:  
Power law indexes,  $\alpha_E \sim -1.6$  and  $\alpha_{ET} < 0.55$ , are consistent with  $S = 3$  SOC theory
- Magnetic reconnection at the surface of accretion flow is likely the origin of X-ray flares
- We propose an MHD model for flares by analogy with solar flares/CME, motivated by the statistical results
- Our model reasonably explains the observed flare light curves and SED

Thank you!