True CMB Power Spectrum Estimation

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Abstract

The cosmic microwave background (CMB) power spectrum is a powerful cosmological probe as it entails almost all the statistical information of the CMB perturbations. Having access to only one sky, the CMB power spectrum measured by our experiments is only a realization of the true underlying angular power spectrum. In this paper we aim to recover the true underlying CMB power spectrum from the one realization that we have without a need to know the cosmological parameters. The CMB power spectrum is very sparse in two dictionaries; Discrete Cosine Transform (DCT) and Wavelet Transform (WT), and can be recovered with only a few percentage of the coefficients. Using the two dictionaries we develop a technique that estimates the true underlying CMB power spectrum from data alone, i.e. without a need to know the cosmological parameters.

This smooth estimated spectrum can be used to simulate CMB maps with similar properties to the true CMB simulations with the correct cosmological parameters. The developed IDL code, **TOUSI**, for Theoretical pOwer spectrUm using Sparse estImation, will be released with the next version of ISAP[a].

[a] \http://jstarck.free.fr/isap.html

GOAL

Using sparsity of the CMB power spectrum we develop a technique that estimates the true underlying CMB power spectrum from data alone; hence without a need to know the cosmological parameters.

Sparse CMB Power Spectrum - TOUSI Algorithm

A signal X considered as a vector in $\mathbb{R}^N$, is sparse if most of its entries are equal to zero. Generally signals are not sparse in direct space, but can be sparsified by transforming them to another domain; for eg, $\sin(x)$ is 1-sparse in the Fourier domain. In the so-called sparsity synthesis model, a signal can be represented as the linear expansion

$$X = \Phi\alpha = \Sigma_{i=1}^{T}\phi_i\alpha[i] \ , \qquad (1)$$

where $\alpha[i]$ are the synthesis coefficients of X and ϕ_i are called the atoms (elementary waveforms) of the dictionary $\Phi = (\phi_1, \ldots, \phi_T)$; such as Fourier (FT), wavelet (WT) and discrete cosine transforms (DCT).

- The CMB power spectrum is very sparse in both the DCT and WT dictionaries, although their sparsifying capabilities are different; DCT recovers global features of spectrum (i.e. the peaks and troughs) while WT recovers localized features.

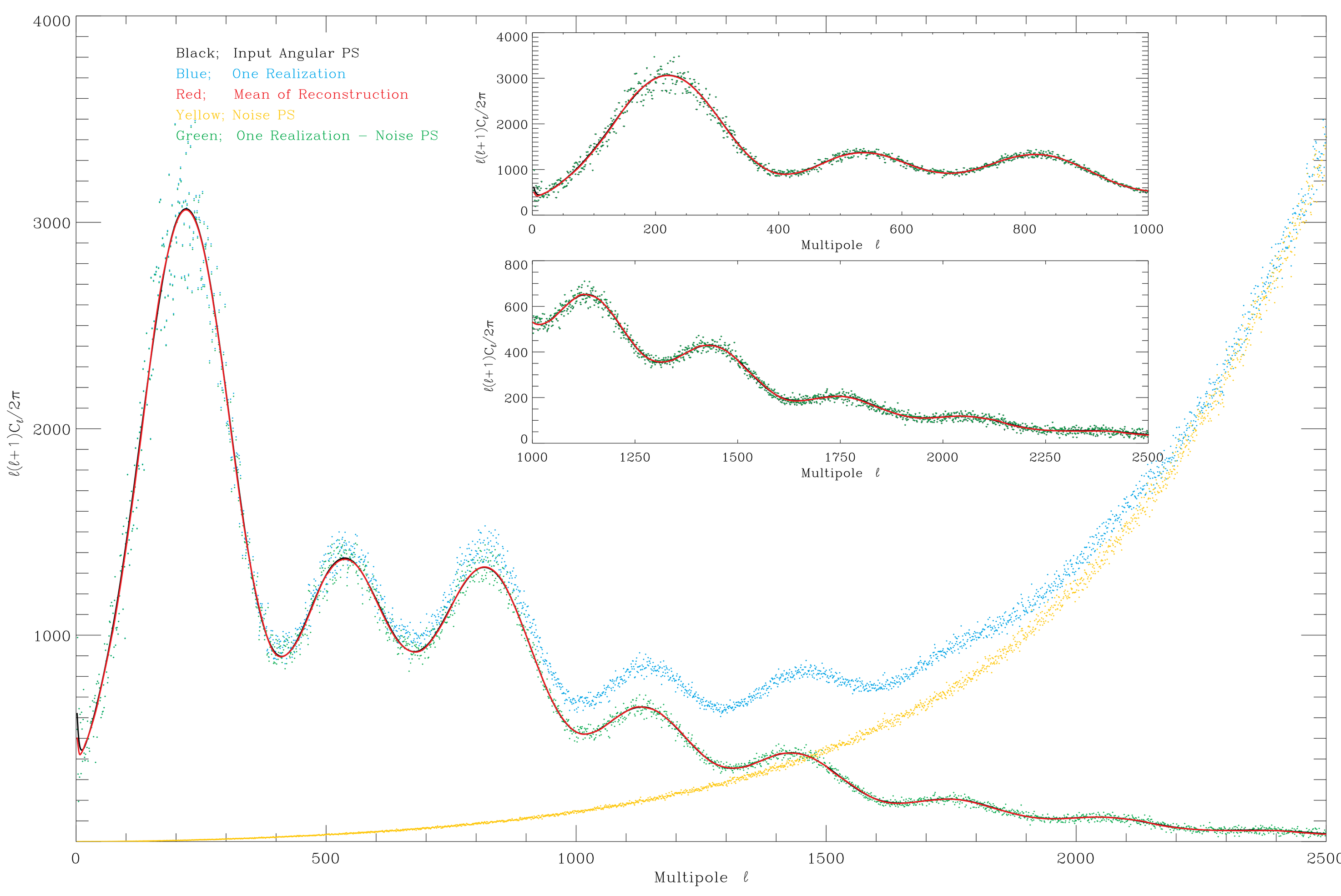
These complementary capabilities of DCT and WT transforms are combined to propose a versatile way for adaptively estimating the theoretical power spectrum from a single realization. Having $X = C[\ell]$ and $\bar{S}_N[\ell]$ as the noise power spectrum, the equation to minimize is

$$\min_X \|\Phi^T X\|_1 \quad \text{s.t.} \quad \begin{cases} X \geqslant 0 \\ M_d \odot (\Phi_d^T \mathcal{T}(X + \bar{S}_N)) = M_d \odot (\Phi_d^T C^s), \ d \in \{1, \cdots, D\} \end{cases} \qquad (2)$$

which is performed iteratively;

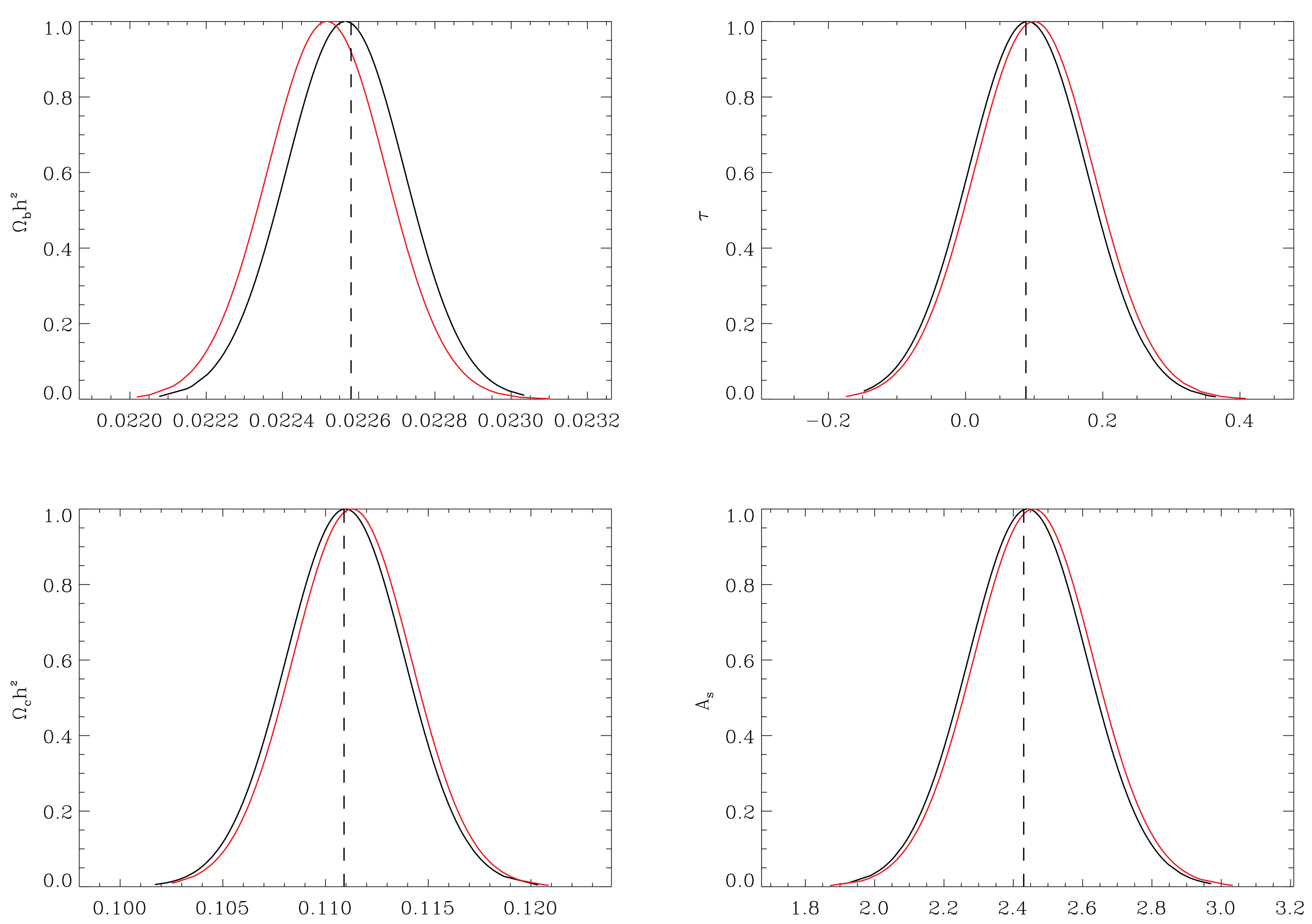
$$\begin{aligned} \tilde{X} &= \mathcal{R}\left(\mathcal{T}\left(X^{(n)} + \bar{S}_N\right) + \Phi M \odot \left(\Phi^T\left(C^s - \mathcal{T}\left(X^{(n)} + \bar{S}_N\right)\right)\right)\right) - \bar{S}_N \\ X^{(n+1)} &= \mathcal{P}_+\left(\Phi \ \mathrm{ST}_{\lambda_n}(\Phi^T \tilde{X})\right) \ . \end{aligned} \qquad (3)$$

Recovering the True Power Spectrum



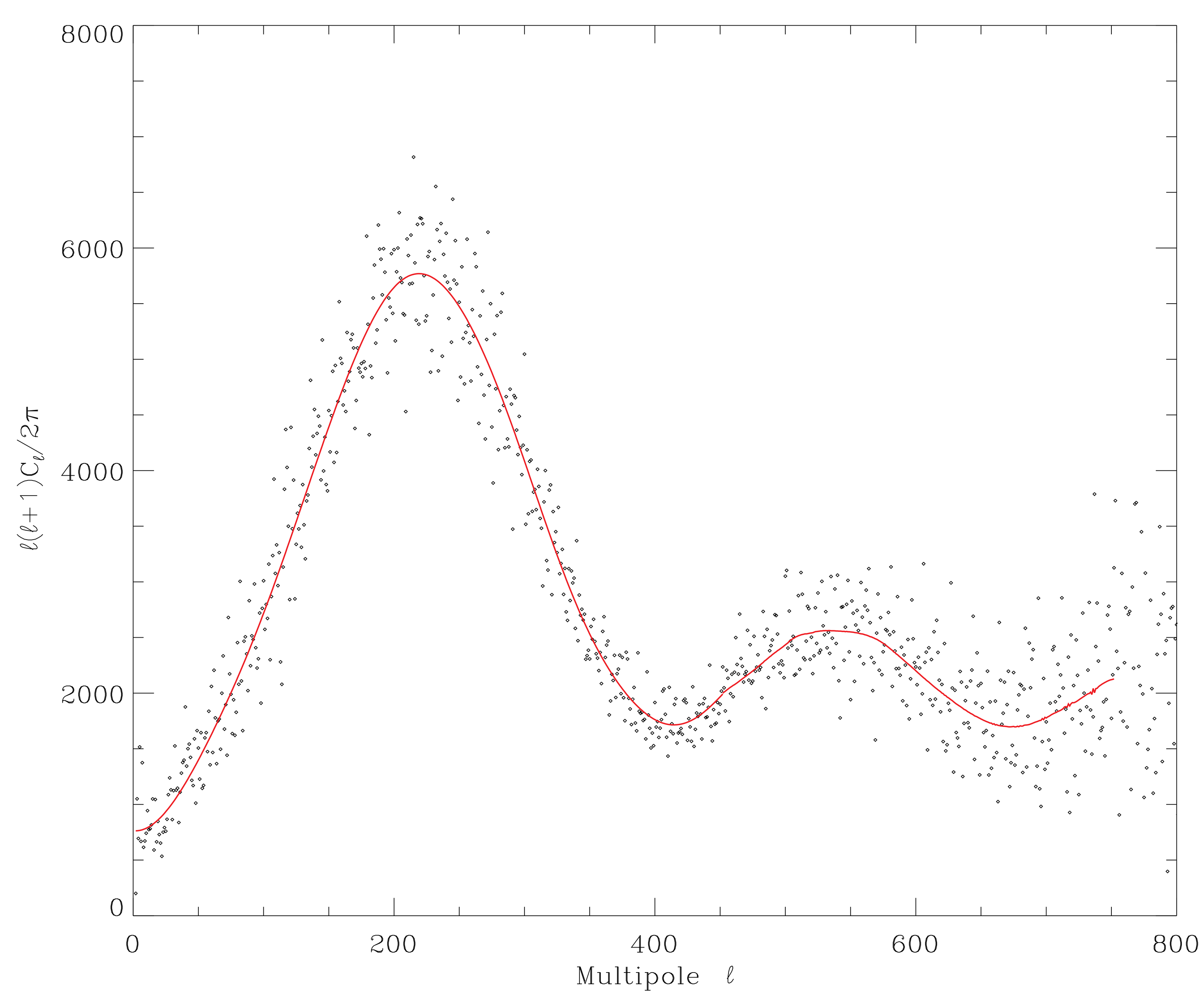
Blue dots show the empirical power spectrum of one realization having instrumental noise. Yellow dots show the estimated power spectrum of one of the simulated noise maps. Green dots show the the spectrum with the noise power spectrum removed. The black and red solid lines are the input and reconstructed power spectra respectively. The inner plots show a zoomed-in version.

Recovering the Cosmological Parameters



Cosmological parameters estimated from the true CMB power spectrum (black line) and the mean of reconstructed power spectra (red line). The dashed line is the true input parameters, i.e. the ones used to calculate the theoretical power spectrum using CAMB.

WMAP7 Power Spectrum



TOUSI works up to $\ell \sim 800$, before features are washed out by the noise.