Constraints on primordial magnetic fields with WMAP7

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ABSTRACT

Magnetism is present at all astrophysical scales. Recent CMB missions as WMAP and Planck provide an interesting tool to improve our knowledge of the Universe and constrain primordial magnetism. To constrain it, we analize its imprint on CMB polarization via Faraday rotation of the polarization plane. Here we present constraints on a stochastic distribution of a magnetic field by using B-modes power spectrum provided by WMAP7 and assuming that a magnetic field is present at the decoupling time. We assume that all E-modes are rotated into B-modes Our results show that a strenght less than 87.6 nanoGauss and a spectral index nB < 1.77 are compatible with WMAP7 B-modes data.

A stochastic distribution of PMF

Magnetic fields are present at every scale in the Universe. For galaxies and clusters of galaxies, the magnetic field strength is between 1 and 10 μG. Moreover, Faraday rotation have been observed at high-redshift (Kronberg et al., 2008) and in Ly-α systems (Oren & Wolfe, 1995) which could indicate a primordial origin of magnetism.

There are several mechanisms to generate primordial magnetic fields. Excellent reviews are Grasso and Rubinstein (2001), Giovannini (2002), Battaner and Florido (2009), Kandus et al. (2010) and references therein.

In this work, we assume that the magnetic field has been generated at some pre-decoupling epoch. We are not interested in the mechanism but only in the observable effects of this primordial magnetic field (hereafter PMF).

The presence of a PMF at decoupling time is expected to modify the trajectory of photons, so its effects could be observable on the CMB. These effects depend on the initial distribution of the field and the modes of the metric which are perturbed. A complete treatment of scalar, vector and tensor perturbations induced by a stochastic magnetic field distribution is given in Paoletti et al. (2009).

Here we derive the observable effects of this stochastic distribution of PMF via Faraday rotation of E-modes into B-modes.

The description of the stochastic PMF assumes that a random magnetic field could be described by the two-point correlation function in the Fourier space as

$$\langle B_i^*(\mathbf{k}) B_j(\mathbf{k'}) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k'}) P_{ij} P_B(k),$$

being $P_{ij} \equiv \delta_{ij} - \hat{k}_i \hat{k}_j$, the projector in the plane; $\hat{k}_i \equiv k_i / k$ and $P_B(k)$, the power spectrum of the magnetic field. Here no helicity is assumed.

The Fourier transform of the field is

$$B_j(\mathbf{k}) = \int d^3 x e^{i\mathbf{k}\cdot\mathbf{x}} B(\mathbf{x}).$$

The power spectrum of the magnetic field is given by

$$P_B(k) = A k^{n_B}$$

where n_B is the spectral index and A is the amplitude given by (see Kosowsky et al. 2005)

$$A = \frac{(2\pi)^{n_B+5}}{2 k_\lambda^{n_B+3}} \frac{B_\lambda^2}{\Gamma\left(\frac{n_B}{2} + \frac{3}{2}\right)}$$

being $k_\lambda = 2\pi/\lambda$, B_λ the strength of the PMF smoothed on a comoving scale λ by convolving with a gaussian kernel (f_λ = N exp($-x^2 / 2\lambda^2$)). We assume that the spectra is vanish for $k > k_D$. The damping wave number is due to Alfvèn wave damping from photon viscosity. This cut-off wave number is smaller that the Silk damping and it is defined as

$$\left(\frac{k_D}{Mpc^{-1}}\right)^{n_B+5} \approx 2.9{\times}10^4 \left(\frac{B_\lambda^2}{10^{-9} G}\right)^{-2} \left(\frac{k_\lambda}{Mpc^{-1}}\right)^{n_B+3} h.$$

Faraday rotation due to a PMF

The presence of a stochastic PMF in the last scattering surface induces a Faraday rotation of the CMB polarization plane. This signal must be printed in the polarization power spectrum and it arises by becoming E-modes into B-modes, in a first approximation. Cross-correlations of EB and TE must arise too but a more detailed computation has to be implemented.

The angle rotated by the presence of a PMF in the Fourier space is given by

$$\phi = RM\lambda^2 \approx \frac{3}{4(2\pi)^5 \nu_0^2 e} \int d^3 k \mathbf{B}(\mathbf{k}) \cdot \mathbf{n} e^{-i\mathbf{k}\cdot\mathbf{n}\eta_0}$$

Where RM is the rotation measure and η_0 is the conformal time. To compute the RM power spectrum we compute the two-point correlation function and obtain

$$C_l^{RM} \approx \frac{9l(l+1)}{(4\pi)^3 e^2} \frac{B_\lambda^2}{\Gamma(n_B + 3/2)} \left(\frac{\lambda}{\eta_0}\right)^{n_B+3} \int_0^{x_D} dx x^{n_B} j_l^2(x)$$

The power spectrum of the rotated angle is

$$C_l^{\phi} = \nu_0^{-4} C_l^{RM}$$

The detection of PMF via Faraday rotation comes from the rotation of E-modes due to the presence of this PMF into B-modes. The computation of the power spectrum of the B-modes obtain is given by (see Kahniashvili et al. 2009)

$$C_l^{BB} = N_l^2 \sum_{l_1 l_2} \frac{(2l_1+1)(2l_2+1)}{4\pi(2l+1)} N_{l_2}^2 K(l,l_1,l_2)^2 C_{l_2}^{EE} C_{l_1}^{\alpha} (C_{l_1 0 l_2 0}^{l0})^2$$

Where $N_l = (2(l-2)!/(l+2)!)^{1/2}$ s the normalization factor, $K(l,l_1,l_2) = -1/2(L^2 + L_1^2 + L_2^2 - 2L_1 L_2 - 2L_1 L + 2L_1 - 2L_2 - 2L)$

With $L \equiv l(l+1),\ L_1 \equiv l_1(l_1+1),\ L_2 \equiv l_2(l_2+1);$ and $C_{l_1 0 l_2 0}^{l0}$ are the Clebsch-Gordan coefficients.

Method

We simulated B-modes due to Faraday rotation of E-modes for ν = 41, 61 and 94 GHz. Moreover, we consider three different values for the tensor-scalar ratio, r = 1, 0.1, 0.001.

A grid-based approach is defined to explore the parameter space given by the stochastic distribution of the PMF. We consider 40 different values for the field strength between 0.1 and 90 nG and 17 different values for the spectral index between -3 and +3.

To constrain the power spectrum of the B-modes produced by the Faraday rotation due to a stochastic PMF, we perform a χ^2-reduced statistics by comparing with the WMAP7 data for each frequency and r

$$\chi^2 = \frac{1}{N-M} \sum_l \frac{(C_l^{BB}(model) - C_l^{BB}(data))^2}{\sigma^2}$$

where, N is the total number of multipoles, M is the number of free parameters, the B-modes simulated for the PMF distribution are numerically computed for each strength and spectral index values. The observed power spectrum of B-modes provided by WMAP7 gives us the associated error too.

Results

In table 1 we show our results for a grid exploration of the parameter space defined for every model.

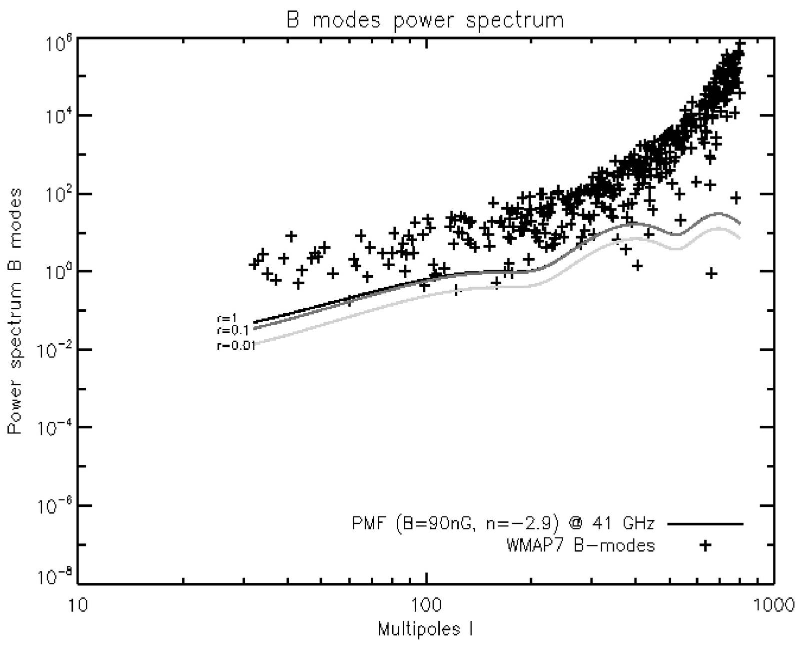
The obtained values of the parameters has a 95 % confidence level.

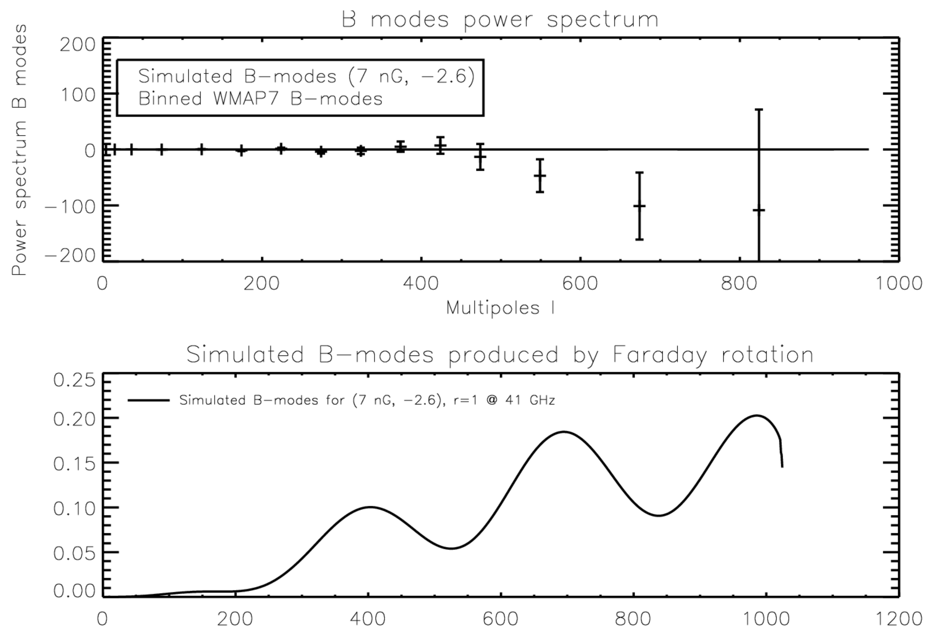
Frequency (GHz)	r	Confidence regions (95%)	χ^2	Reduced-χ^2
41	1	$B_\lambda > 2.50,\ n_B < 1.77$	709.31	0.93
41	0.1	$B_\lambda > 2.50,\ n_B < 1.77$	709.4	0.93
41	0.01	$B_\lambda > 2.50,\ n_B < 1.68$	710.71	0.93
61	1	$B_\lambda < 87.6,\ n_B > -2.77$	791.00	1.03
61	0.1	$B_\lambda < 87.6,\ n_B > -2.77$	791.00	1.03
61	0.01	$B_\lambda < 87.6,\ n_B > -2.77$	791.00	1.03
94	1	$B_\lambda < 87.9,\ n_B > -2.78$	832.74	1.09
94	0.1	$B_\lambda < 87.9,\ n_B > -2.78$	832.74	1.09
94	0.01	$B_\lambda < 87.9,\ n_B > -2.79$	832.74	1.09

Table 2. Best fits for C_l^{BB} for WMAP7 Q, V and W bands. First column shows the frequency; second shows the confidence region for B_λ and n_B; fourth shows χ^2 and the last column, the reduced χ^2 for a non-blind exploration.

Constraints derived from the B-modes binned spectrum are B_λ < 88 nG and n_B > -2.8 for all frequencies and for all values considered for the tensor-scalar ratio.

In Figure 1, we plot the best-fit for 41 GHz in logarithmic scale without error bars. Top panel of figure 2 shows the B-modes binned power spectrum observed with the error associated for these measurments and our best-fit at 41 Ghz. Bottom panel of figure 2 shows the numerical computation of the B-modes power spectrum for our best-fit model of PMF.





Conclusions

We constrain the field strength and the spectral index of a stochastic PMF by using the B-modes provided by WMAP7 for 41, 61 and 94 GHz and assuming that the E-modes becomes in B-modes due to the Faraday rotation induced by this PMF.

The best frequency to constrain the PMF distribution due to Faraday rotation with WMAP7 is 41 GHz. The data requieres a field strength higher than 2.5 nG and a spectral index lower than 1.77. This is compatible with results constraints in Paoletti & Finelli (2011). As indicated in Figure 1, large values field strength of the order of magnitude of 100 nG seems to fit better the observed B-modes and as concluded in Pogosian et al. (2011) and in Kahniashvili et al (2009). This kind of values does not improve the fit because the error associated for each measurement is high specially at larger multipoles. In Figure 2, we plot the binned B-modes power spectrum provided by the WMAP team with our best fit. Results for the binned power spectrum or without binning are compatible, a field strength between 2.5 and 88 nG and a spectral index between -2.8 and 1.8 is required.

Our main conclusion is that an important effort with high accuracy in the detection of B-modes is expected with the Planck data. At 30 and 44 GHz, we must expect to detect the Faraday rotation due to a PMF as described here.

References

Battaner, E., & Florido, E. 2009, IAU Symposium, 259,529; Kronberg, P. P., Bernet, M. L., Miniati, F., Lilly, S. J., Short, M. B., & Higdon, D. M. 2008, ApJ, 676, 70 Oren, A. L., & Wolfe, A. M. 1995, ApJ, 445, 624 Kandus, A., Kunze, K. E., & Tsagas, C. G. 2010, arXiv:1007.3891 Giovannini, M. 2002, arXiv:hep-ph/0208152; Grasso, D., & Rubinstein, H. R. 2001, Phys. Rep., 348, 163; Kahniashvili, T., Maravin, Y., & Kosowsky, A. 2009, Phys.Rev.D, 80, 023009; Kosowsky, A., Kahniashvili, T., Lavrelashvili, G., & Ratra, B. 2005, Phys.Rev.D, 71, 043006; Paoletti, D., Finelli, F., & Paci, F. 2009, MNRAS, 396, 523; Paoletti & Finelli, 2011, Phys.Rev.D,83, 123533; Pogosian et al., 2011, Phys.Rev.D, 84, 043530.

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