Information field theory

turning data into images

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für Astrophysik

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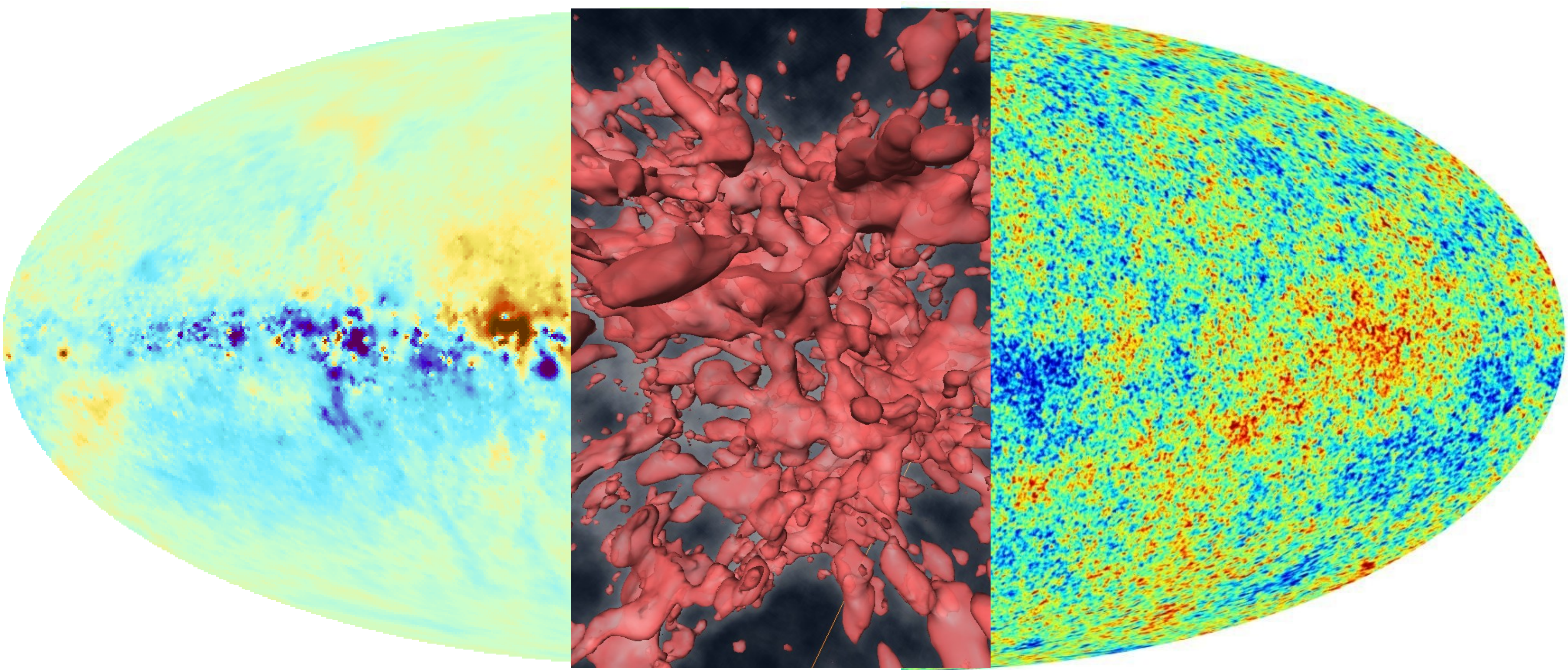


signal fields (CMB, LSS, …): ∞ degrees of freedom
data sets (timelines, galaxy counts, …): finite
→ additional information needed

Corollary:
All imaging algorithms build on prior assumptions to regularize the reconstruction.

Why not to start directly with the assumptions?
space is continuous → information field theory

Information Theory

$s = signal$
$d = data$

posterior

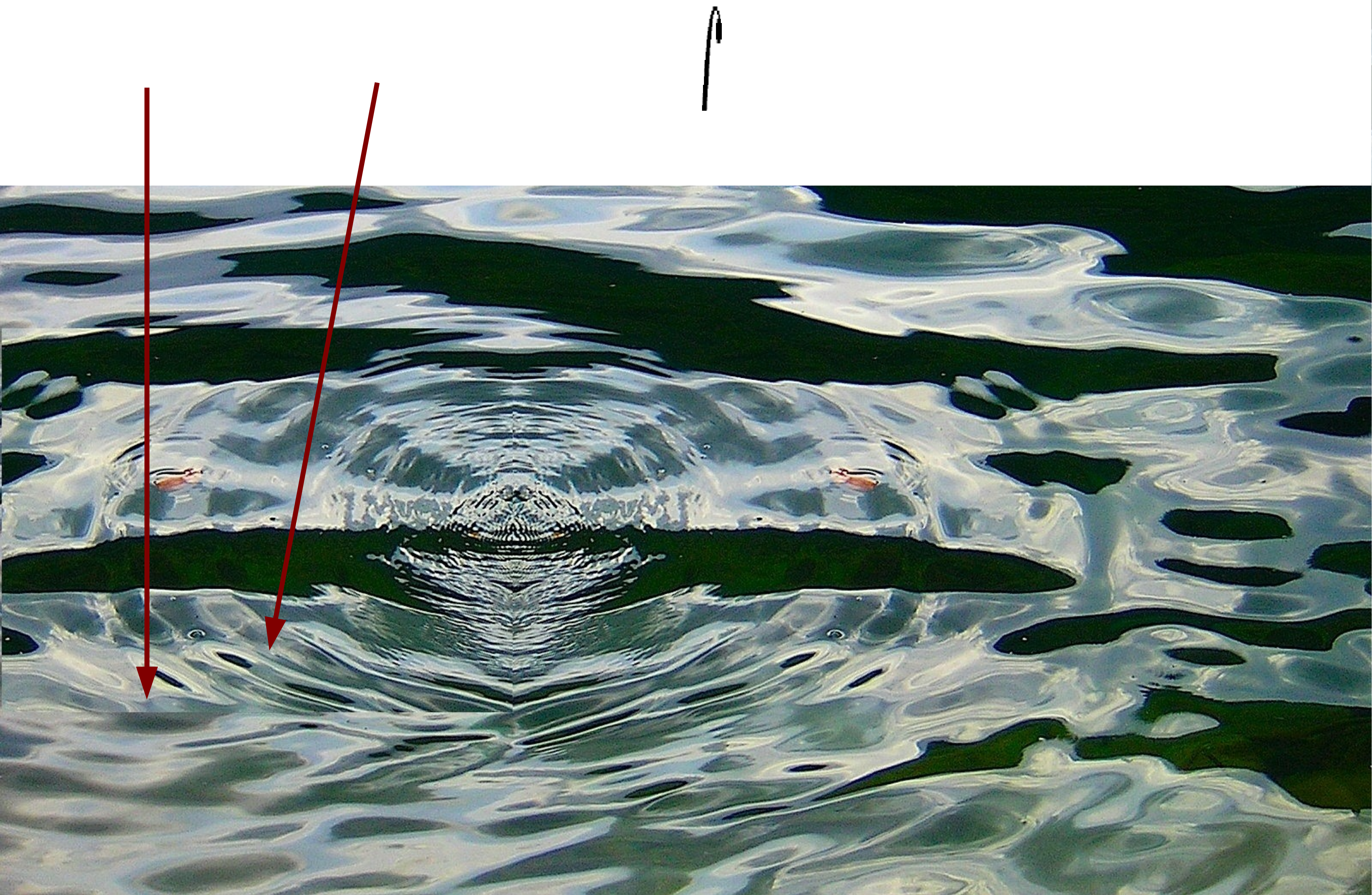
likelihood

prior

$$P(s|d) = \frac{P(d|s)\,P(s)}{P(d)}$$

evidence

inference problem as information field theory



$$\langle s(x_1) \cdots s(x_n) \rangle_{P(s|d)} = \int \mathcal{D}s \, s(x_1) \cdots s(x_n) \, P(s|d)$$

$$\int \mathcal{D}s = \prod_{i=1}^{N_{\mathrm{pix}}} \int ds_i$$

Free Theory

Gaussian signal & noise, linear response

signal :

$$P(s) = \mathcal{G}(s,S) = \frac{1}{|2\pi\, S|^{1/2}} \exp\left(-\frac{1}{2}\, s^\dagger S^{-1} s\right)$$

$$j^\dagger s = \int dx\, j^*(x) s(x)$$

$$S = \langle s\, s^\dagger \rangle_{(s)}$$

data :

$$d = R\, s + n,\; P(d|s) = P(n = d - R\, s)$$

noise :

$$P(n) = \mathcal{G}(n,N),\; N = \langle n\, n^\dagger \rangle_{(n)}$$

Wiener filter theory

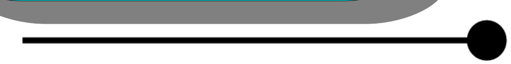
known for 60 years

$$
\begin{aligned}
H(s) &= -\log P(d,s) = -\log P(d|s) - \log \\
&= \frac{1}{2}(d - R\,s)^\dagger N^{-1}(d - R\,s) + \frac{1}{2}\,s^\dagger S^{-1} s + \text{const} \\
&= \frac{1}{2}\,s^\dagger \underbrace{\left(S^{-1} + R^\dagger N^{-1} R\right)}_{\equiv D^{-1}}\,s + s^\dagger \underbrace{R^\dagger N^{-1} d}_{\equiv j} + \text{const} \\
&= \frac{1}{2}\,s^\dagger D^{-1}\,s + s^\dagger j + H_0
\end{aligned}
$$

information source

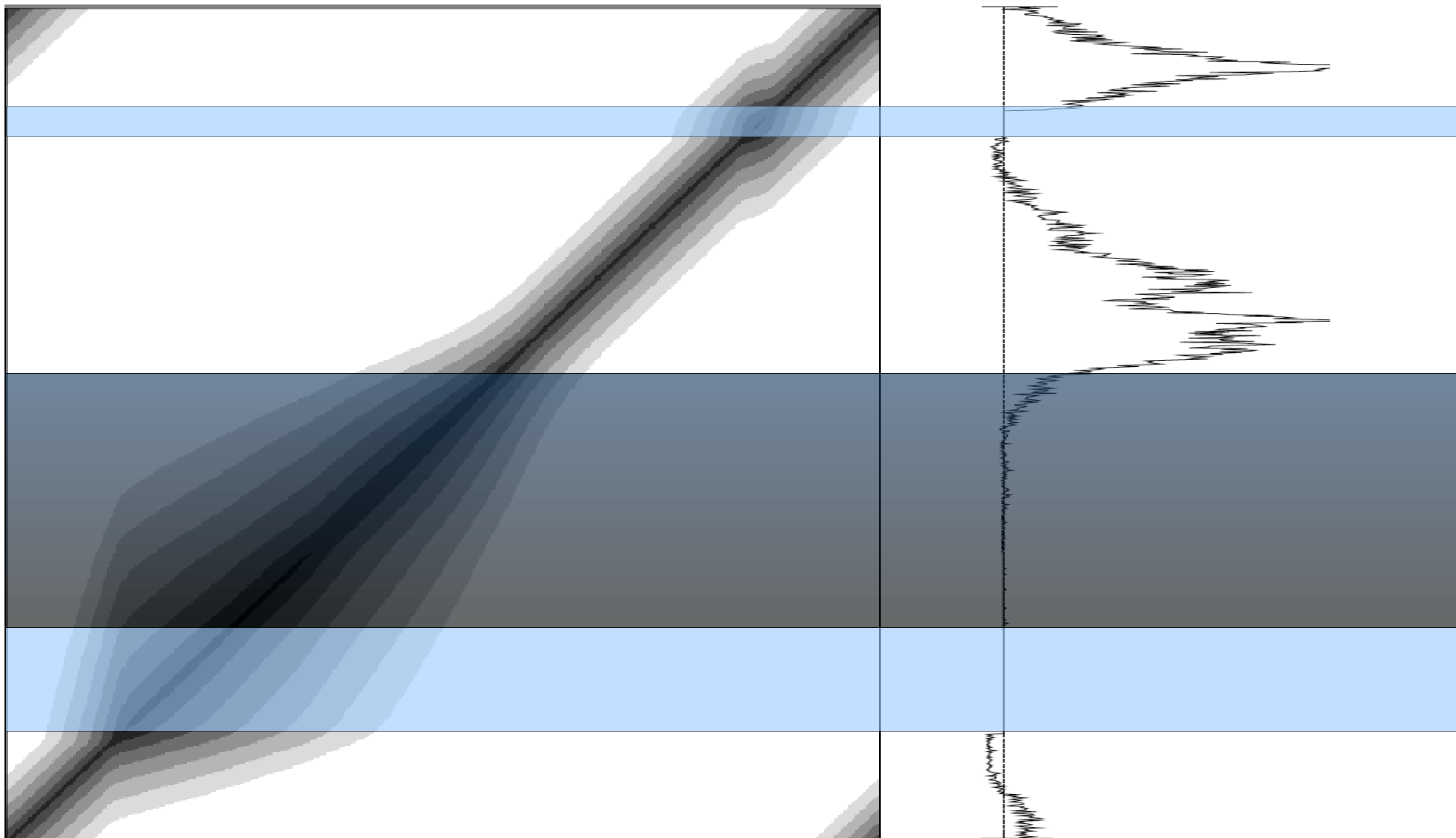
information propagator

$$\text{mean: } m = \langle s \rangle_{(s|d)} = D\,j =$$



$$\text{uncertainty: } \langle (s-m)\,(s-m)^\dagger \rangle_{(s|d)} = D$$

$$m = \langle s \rangle_{(s|d)} = D\, j = D_{xy}\, j_y$$



$$j = R^\dagger N^{-1} d$$

$$D = \left[S^{-1} + R^\dagger N^{-1} R\right]^{-1}$$

Interacting Theory

non-Gaussian signal, noise, or non-linear response

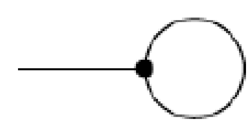
$$H[s] = \frac{1}{2}\, s^\dagger D^{-1}\, s - j^\dagger s + H_0 + \sum_{n=3}^{\infty} \frac{1}{n!}\, \Lambda^{(n)}_{x_1 \dots x_n}\, s_{x_1} \cdots s_{x_n}$$

Taylor-Fréchet expansion of Hamiltonian

Use expansion into Feynman diagrams

$\langle s \rangle_{(s|d)} =$

$+$



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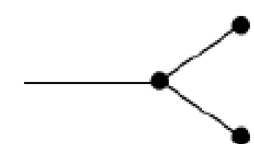
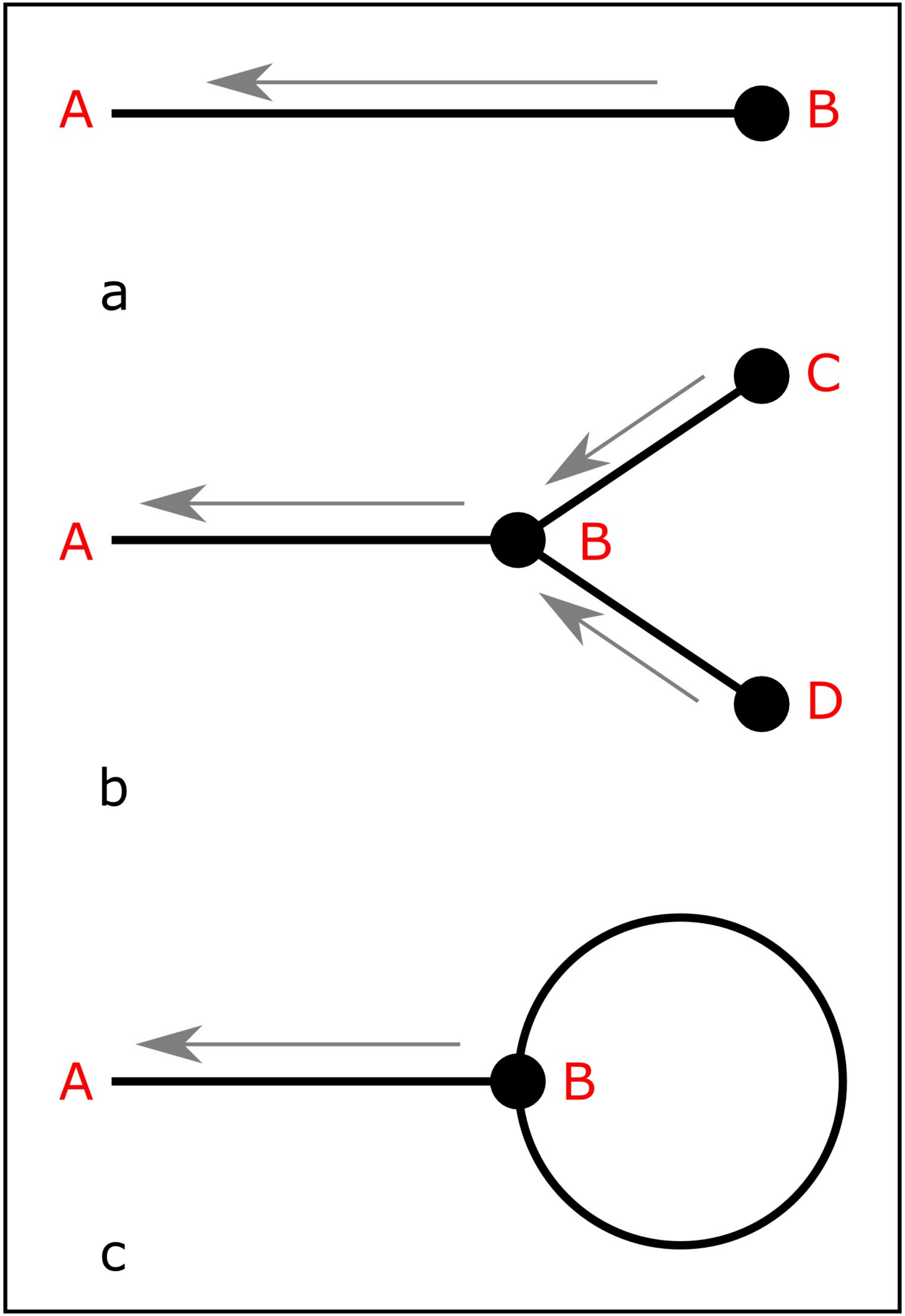


$+ \quad \dots$

$$= D_{xy}\, j_y \; - \frac{1}{2} D_{xy}\, \Lambda^{(3)}_{yzu}\, D_{zu}$$

$$- \frac{1}{2} D_{xy}\, \Lambda^{(3)}_{yuz}\, D_{zz'}\, j_{z'}\, D_{uu'}\, j_u \quad + \quad \dots$$


A
B
a
C
A
B
D
b
A
B
c

IFT dictionary

Translation:

inference problem → statistical field theory

$$P(s|d) = \frac{P(d|s)\,P(s)}{P(d)} \equiv \frac{1}{Z}\,e^{-H[s]}$$

Dictionary:

log-Posterior	=	negative Hamiltonian
evidence	=	partition function Z
Wiener variance	=	information propagator
noise weighted data	→	information source
inference algorithms	←	Feynman diagrams
maximum a Posteriori	=	classical solution
uncertainty corrections	=	loop corrections
Shannon information	=	negative entropy

Cosmography

reconstruction of the cosmic large scale structure

- galaxies trace dark matter density field
- log-normal density field: $\varrho = \varrho_0 \, e^{c\, s}$
- galaxy shot noise due to Poisson statistics
- inhomogeneous observation

Hamiltonian:

data = galaxy counts

galaxy bias

$$H[s] = \frac{1}{2}\, s^\dagger S^{-1} s - b\, d^\dagger s + \kappa_0\, e^{b\, s}$$

covariance of Gaussian log-density field *s*

expected mean galaxy counts

Enßlin, Frommert, Kitaura (2009)

classical & renormalized solution

fix point solution for different temperatures T

$T=0$ classical solution, $T=1$: field theoretical solution

$$m = b\,S\,\left(d - \kappa_{b\,m+T\,b\,\widehat{D}/2}\right),$$

$$D = \left(S^{-1} + \widehat{\kappa}_{b\,m+T\,b\,\widehat{D}/2}\right)^{-1}$$

$$\kappa_s = \kappa\, e^{b\,s}$$

non-linear reconstruction

Jasche, Kitaura, Li, Enßlin (2010)

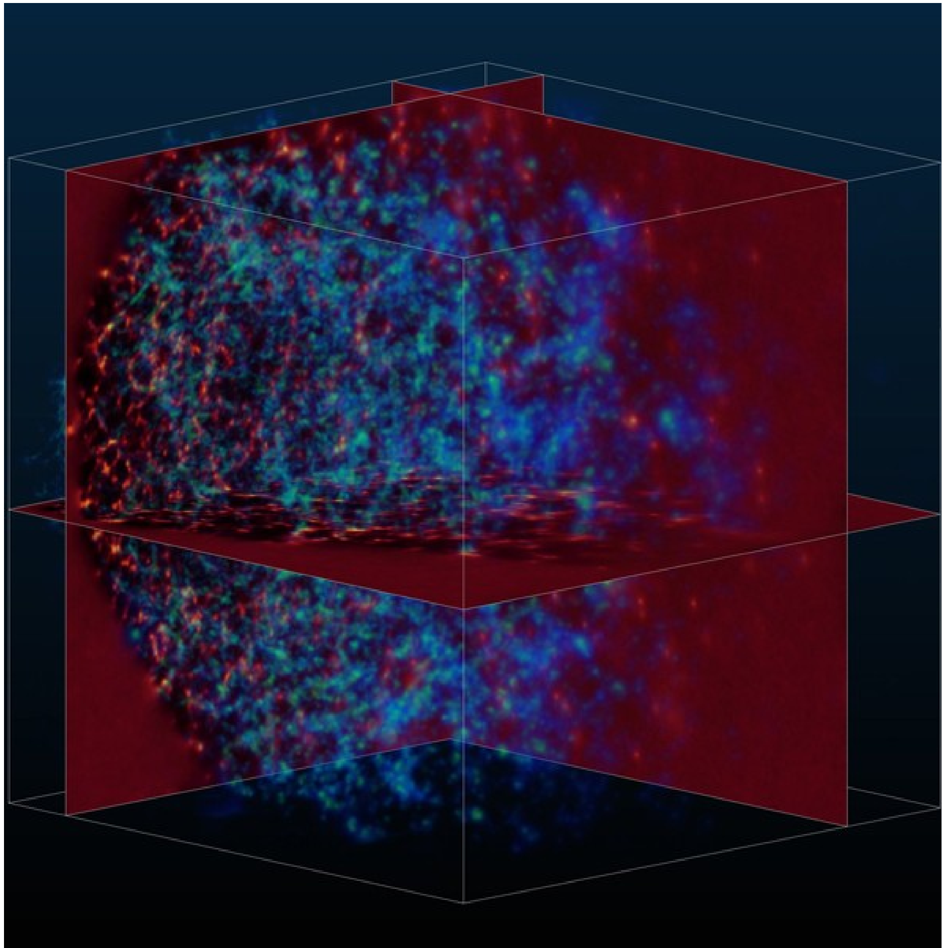
log-density from SDSS DR7 using Hamiltonian sampling

main sample galaxies, 0.001< z < 0.4, ¼ sky

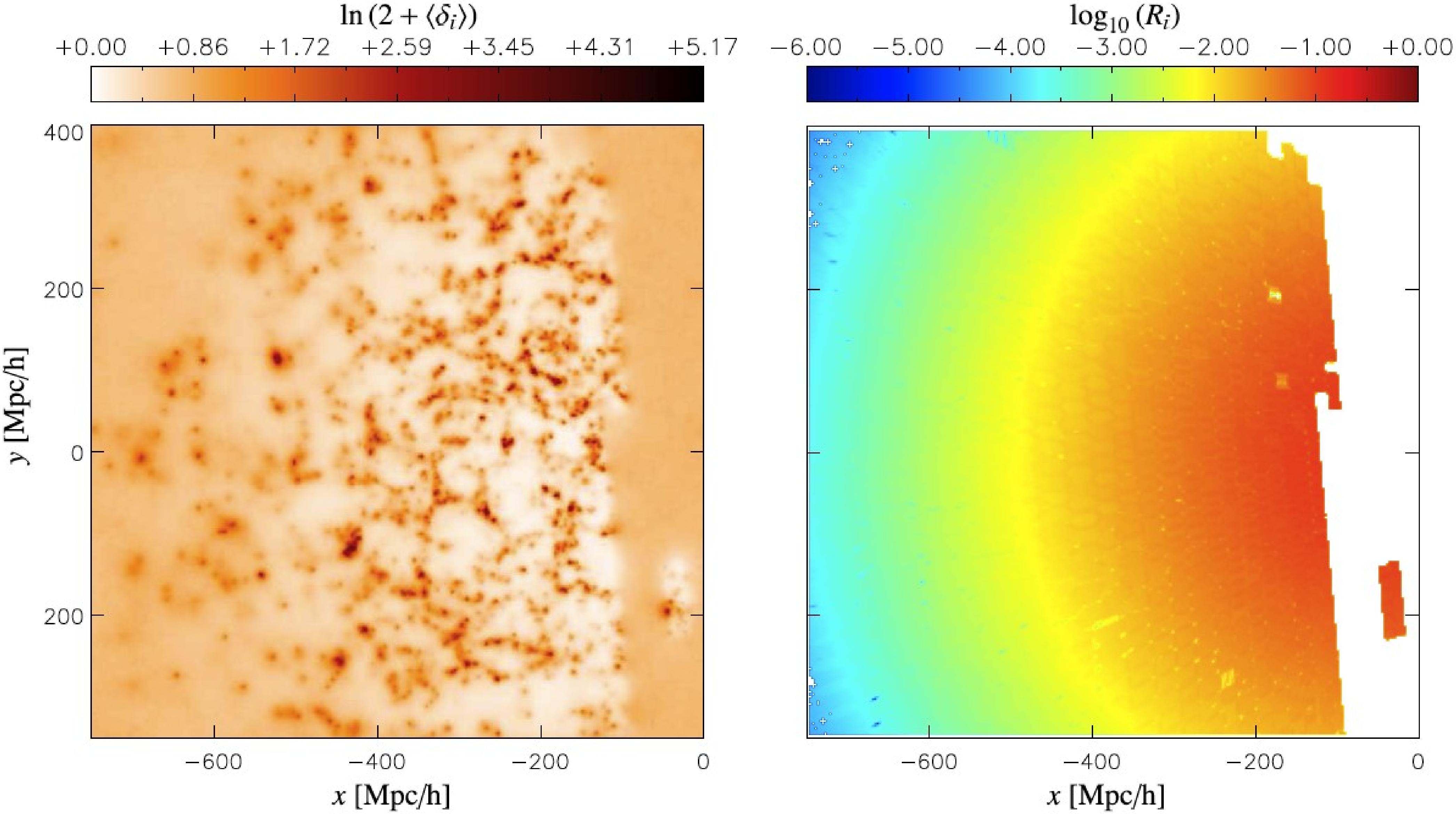
box : 750 Mpc/h per side,
256 voxels per side
→ resolution ~ 3 Mpc/h

corrected for

- selection function
- completeness
- full shot noise

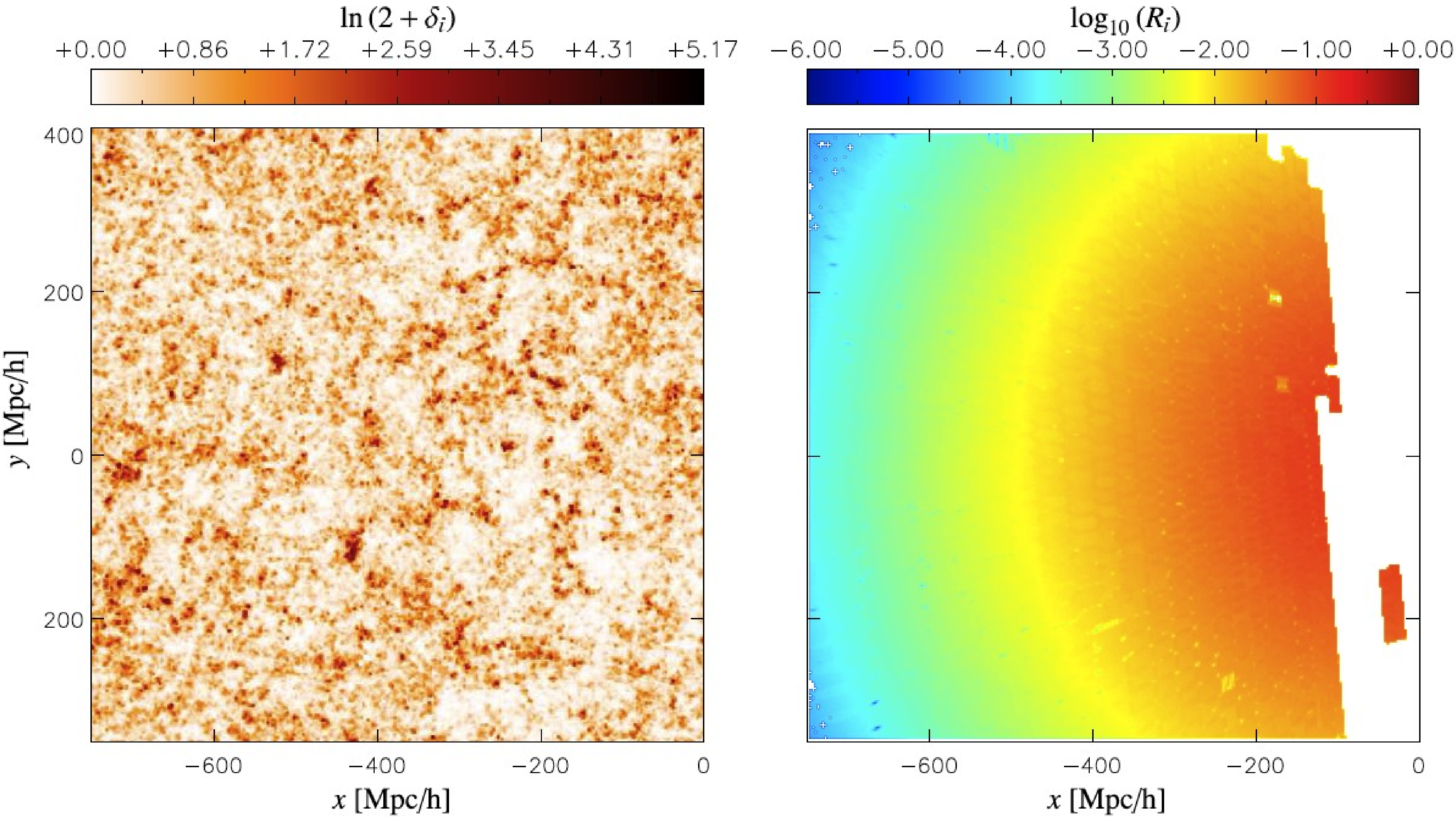


mean + mask



$\ln(2+\langle\delta_i\rangle)$
+0.00 +0.86 +1.72 +2.59 +3.45 +4.31 +5.17
$\log_{10}(R_i)$
−6.00 −5.00 −4.00 −3.00 −2.00 −1.00 +0.00
400
200
0
200
y [Mpc/h]
−600 −400 −200 0
x [Mpc/h]
−600 −400 −200 0
x [Mpc/h]

sample + mask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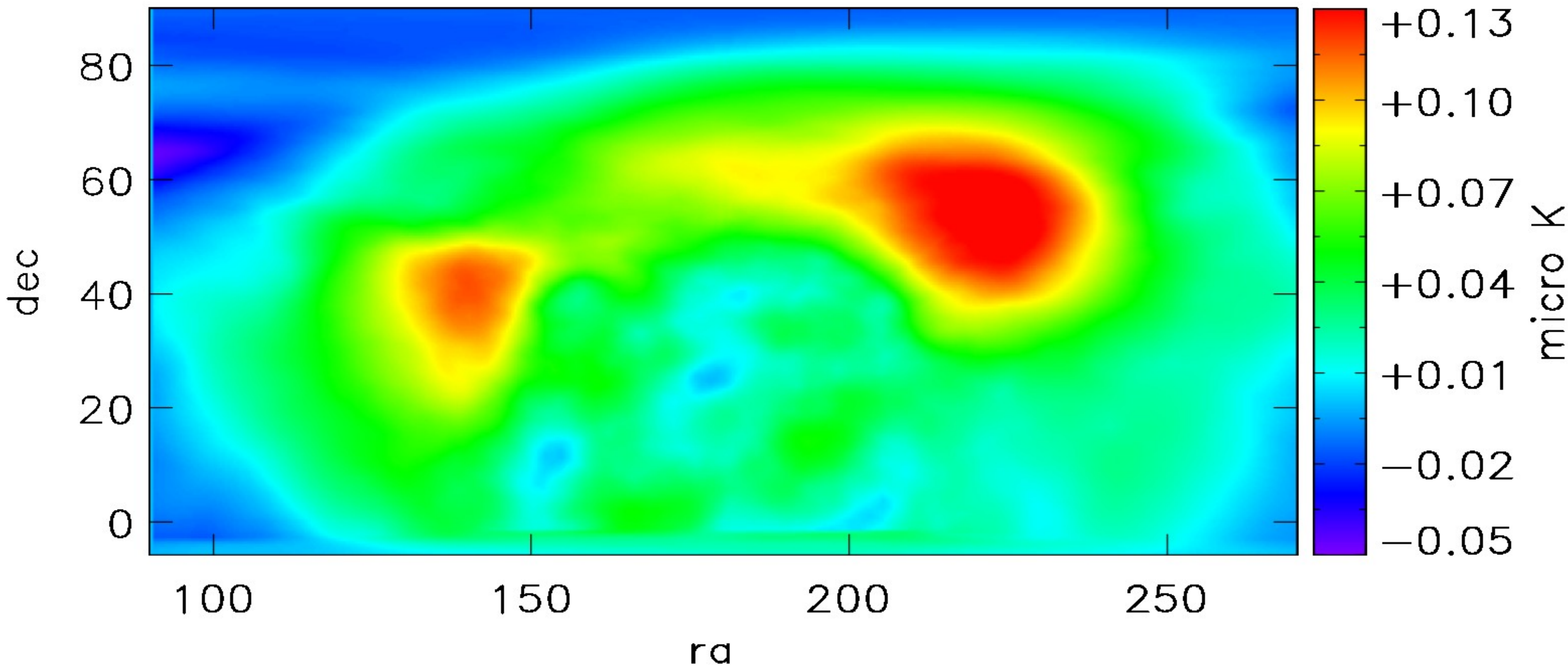
400
200
0
200
y [Mpc/h]
−600 −400 −200 0
x [Mpc/h]
−600 −400 −200 0
x [Mpc/h]

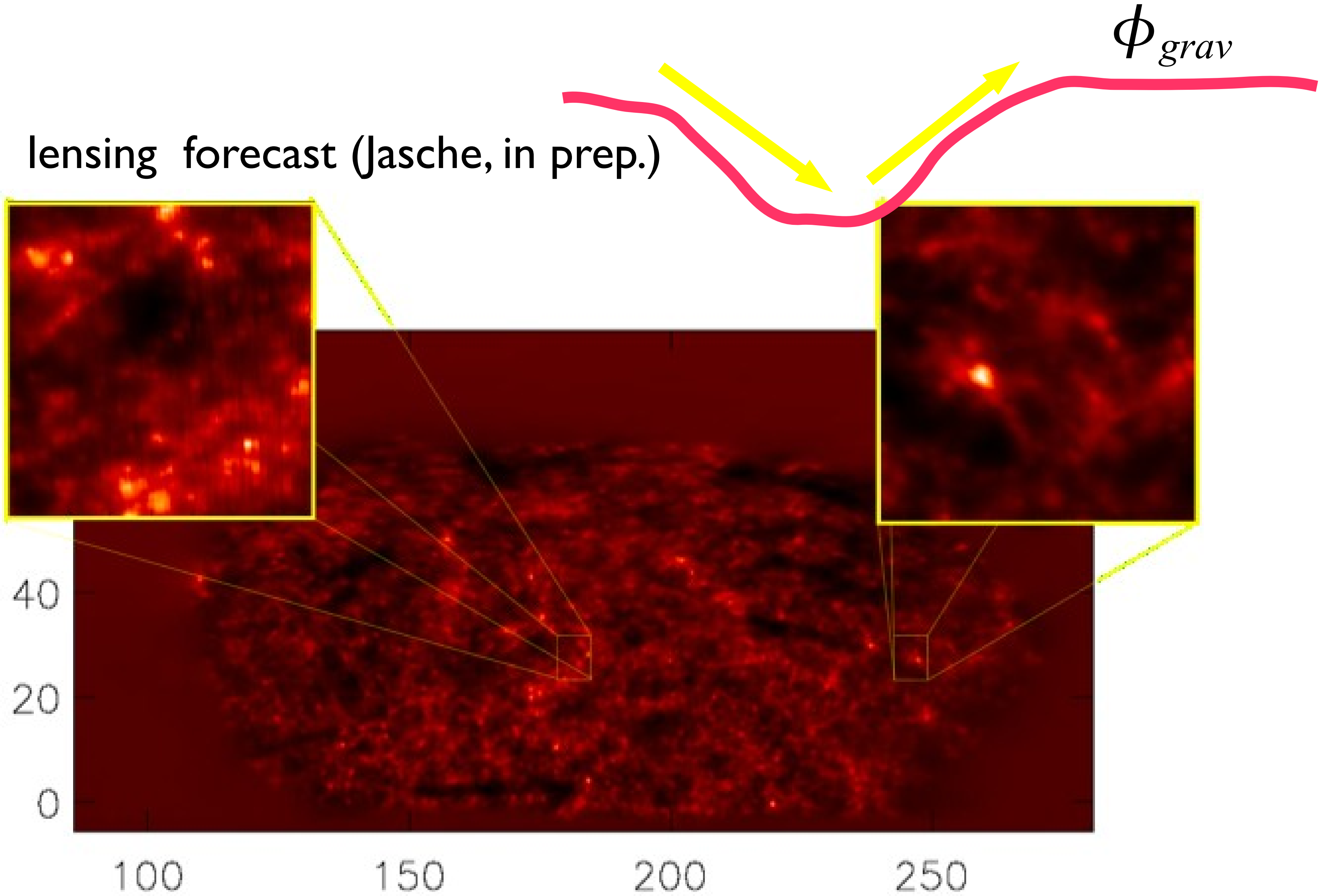
Integrated Sachs Wolfe Effect

ISW forecast (Jasche, in prep.)

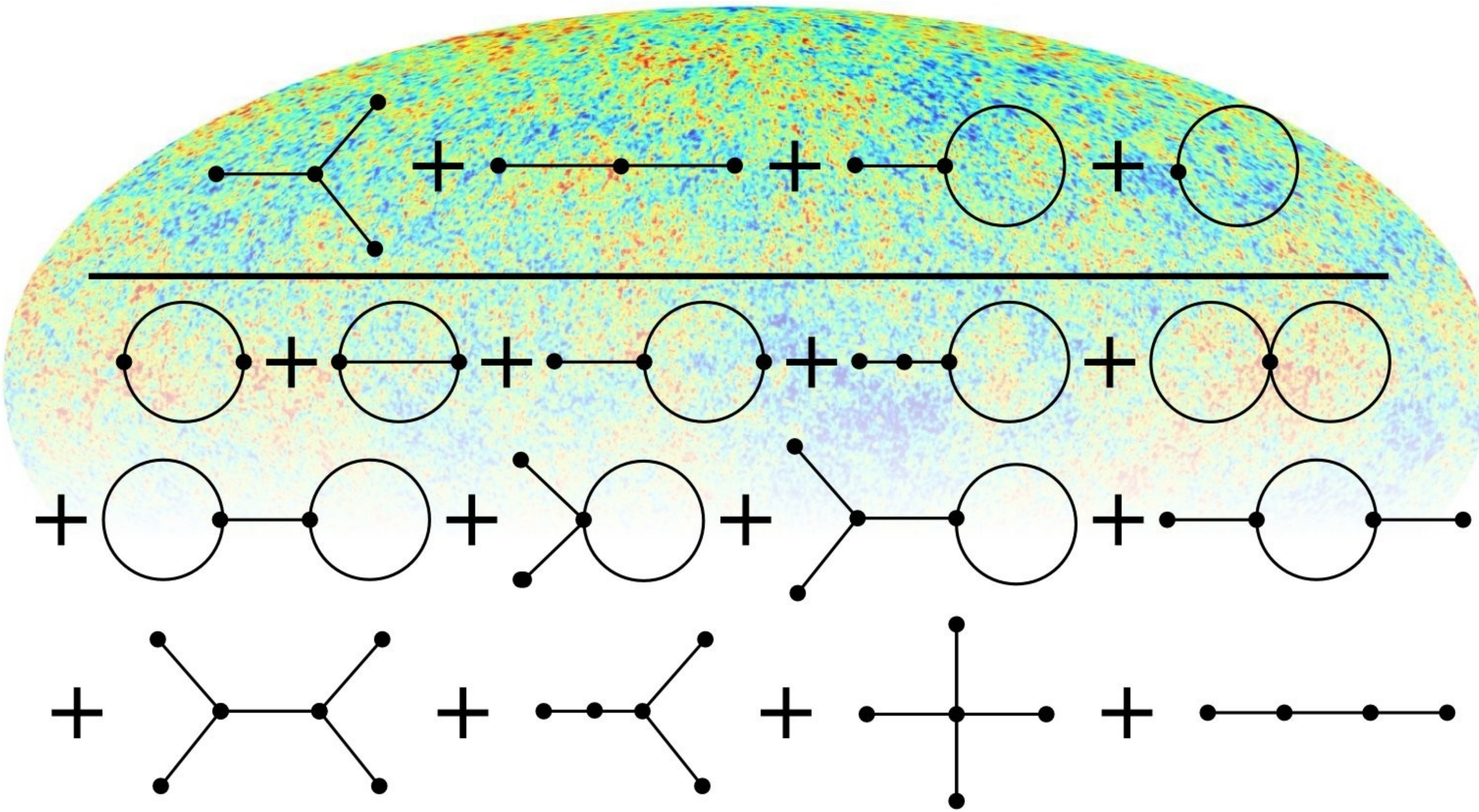


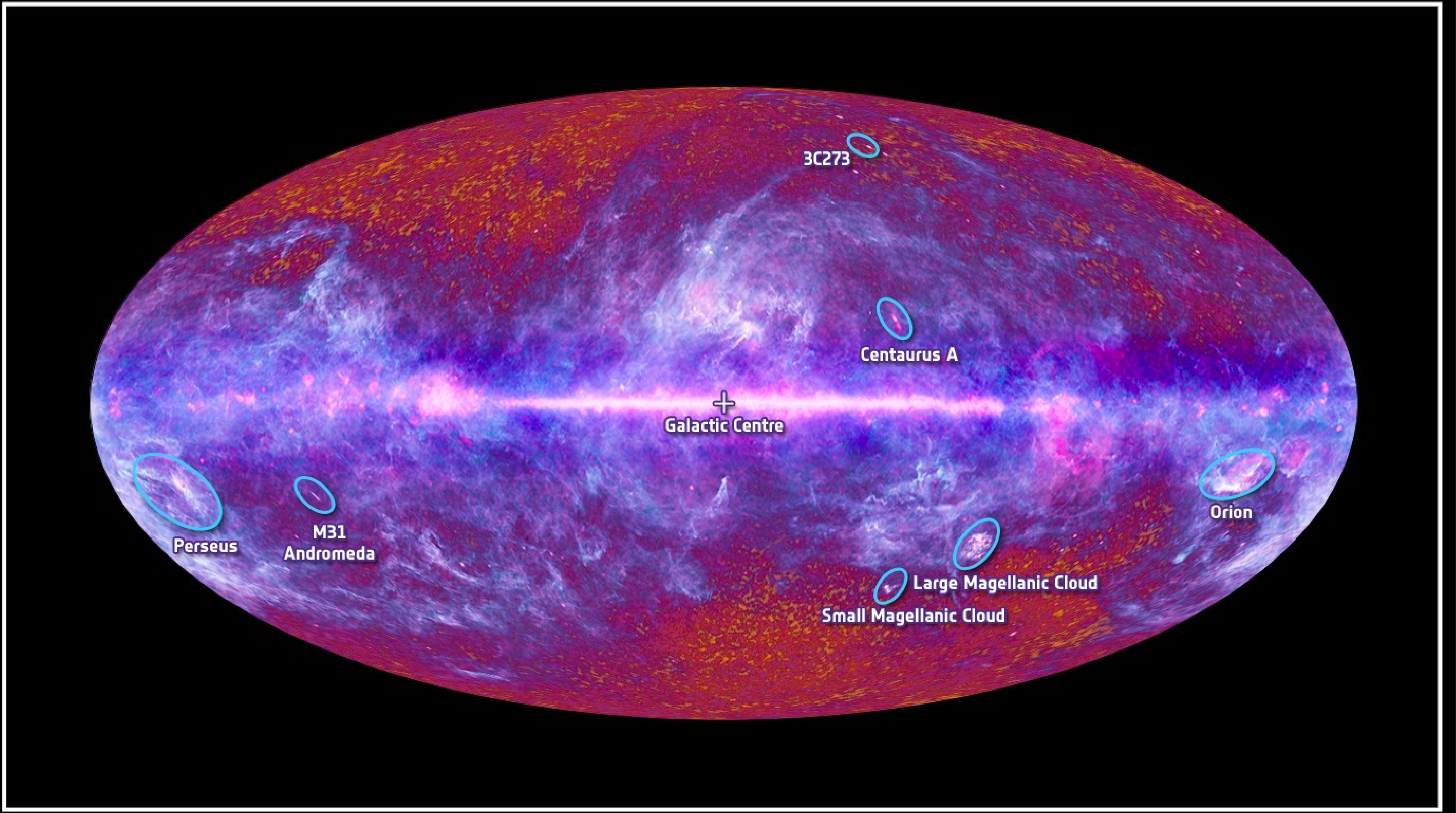
weak gravitational lensing

lensing forecast (Jasche, in prep.)



Primordial non-Gaussianity detection

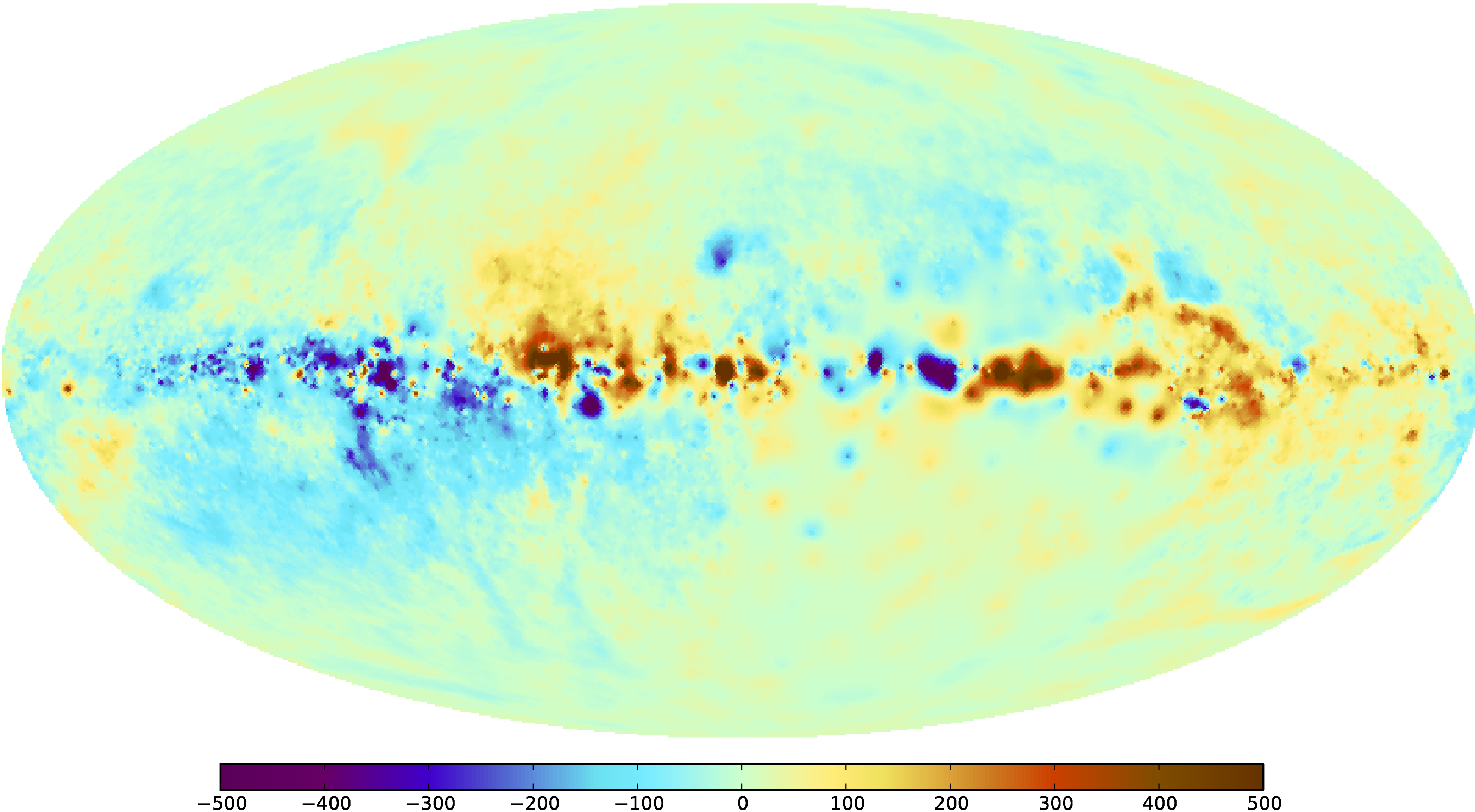




The Planck one-year all-sky survey

Faraday sky

Oppermann et al. (arXiv:1111.6186)



Conclusions

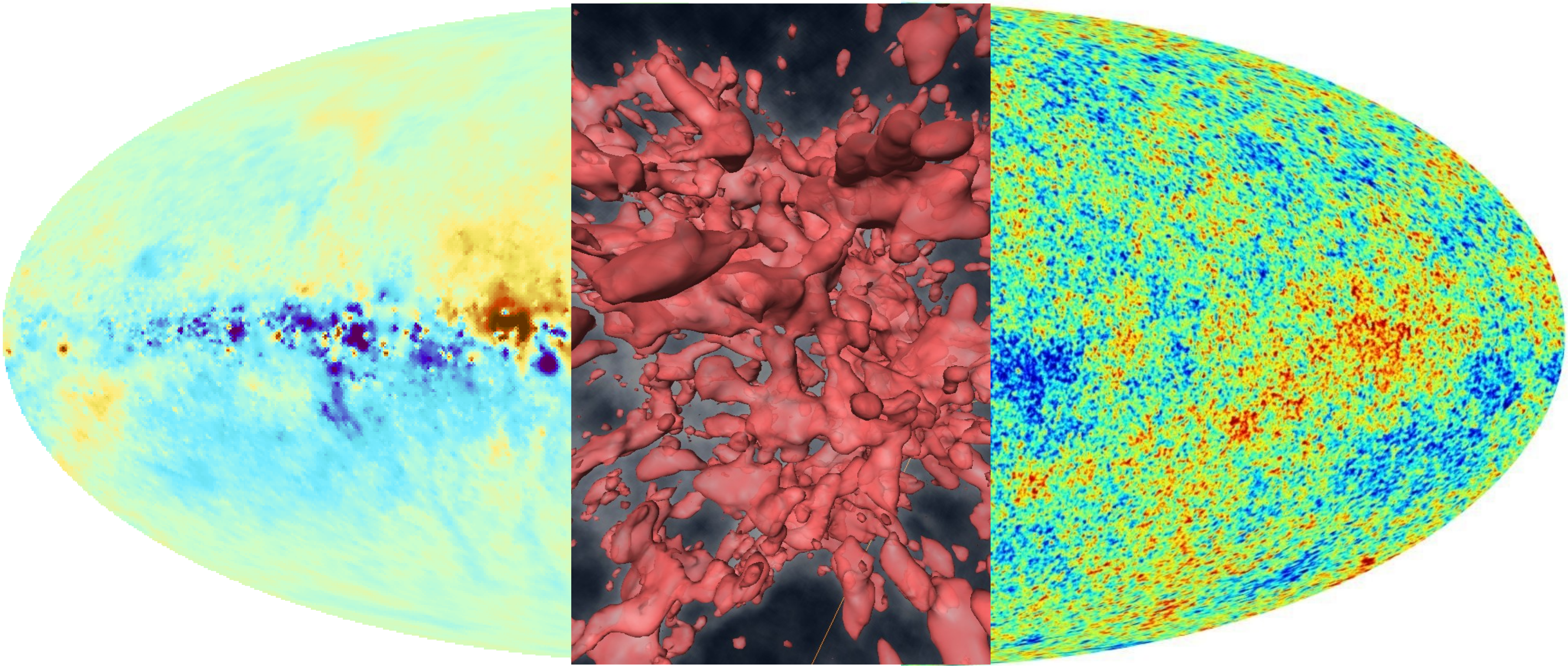
Information field theory (IFT):

- IFT is a mathematical language for field inference
- field reconstruction requires prior = regularization
  e.g. Gaussianity, symmetries, ...
- field theoretical toolbox: free theory = Wiener filter theory, classical theory = max. a posteriori, Feynman diagrams, renormalisation flow, thermodynamical inference, critical filter methodology ...

Cosmography & CMB science:

- clear assumptions (stat. homogeneity & isotropy, power, noise)
  → unique, high fidelity IFT algorithm
- LSS reconstruction, ISW detection, CMB non-Gaussianity

Thank you !



www.mpa-garching.mpg.de/ift